Applications of Calculus I

Application of Maximum and Minimum Values and Optimization to Engineering Problems

by

Dr. Manoj Chopra, P.E.

UCF EXCEL

Outline

- Review of Maximum and Minimum Values in Calculus
- Review of Optimization
- Applications to Engineering
- My Current Research Projects (Potential EXCEL URE Opportunities)

Maximum and Minimum Values

- You have seen these in Chapter 4
- Some important applications of differential calculus need the determination of these values
- Typically this involves finding the maximum and/or minimum values of a Function
- Two Types – Global (or Absolute) or Local (or Relative).

Local Maxima or Minima

- Fermat's Theorem – If a function *f(x)* has a local maximum or minimum at c, and if *f'(c)* exists, then

$$f'(c) = 0$$

- Critical Number *c* of a function *f (x)* is number such that either

$$f'(c) = 0$$

or it does not exist.

Closed Interval Method

- Used to find the Absolute (Global) Maxima or Minima in a Closed Interval [a,b]
  - Find f at the critical number(s) of f in (a,b)
  - Find f at the endpoints
  - Largest value is absolute maximum and smallest is the absolute minimum

First Derivative Test

- Let c be the critical number of a continuous function f:
  - If $f'(c)$ changes from positive to negative at c, then f has a local maximum at c
  - If $f'(c)$ changes from negative to positive at c, then f has a local minimum at c
  - If no change in sign then f has no local extreme values at c

Engineering

ENGINEER IDENTIFICATION TEST

You walk into a room and notice that a picture is hanging crooked. You...

A. Straighten it.
B. Ignore it.
C. Buy a CAD system and spend the next six months designing a solar-powered, self-adjusting picture frame while often stating aloud your belief that the inventor of the nail was not the brightest bulb in the room.

Correct answer is C

He has the Knack!

Engineering - Demo

- http://www.funderstanding.com/k12/coaster/
- Highlights the importance of the following:
  - Understanding of Math
  - Understanding of Physics
  - Influence of Several Independent Variables
  - Fun

Calculus Application – Graphing and Finding Maxima or Minima

Section 4.1 #66:

On May 7, 1992, the space shuttle Endeavor was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite.

The table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

Shuttle Video

Calculus Application – Graphing and Finding Maxima or Minima

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

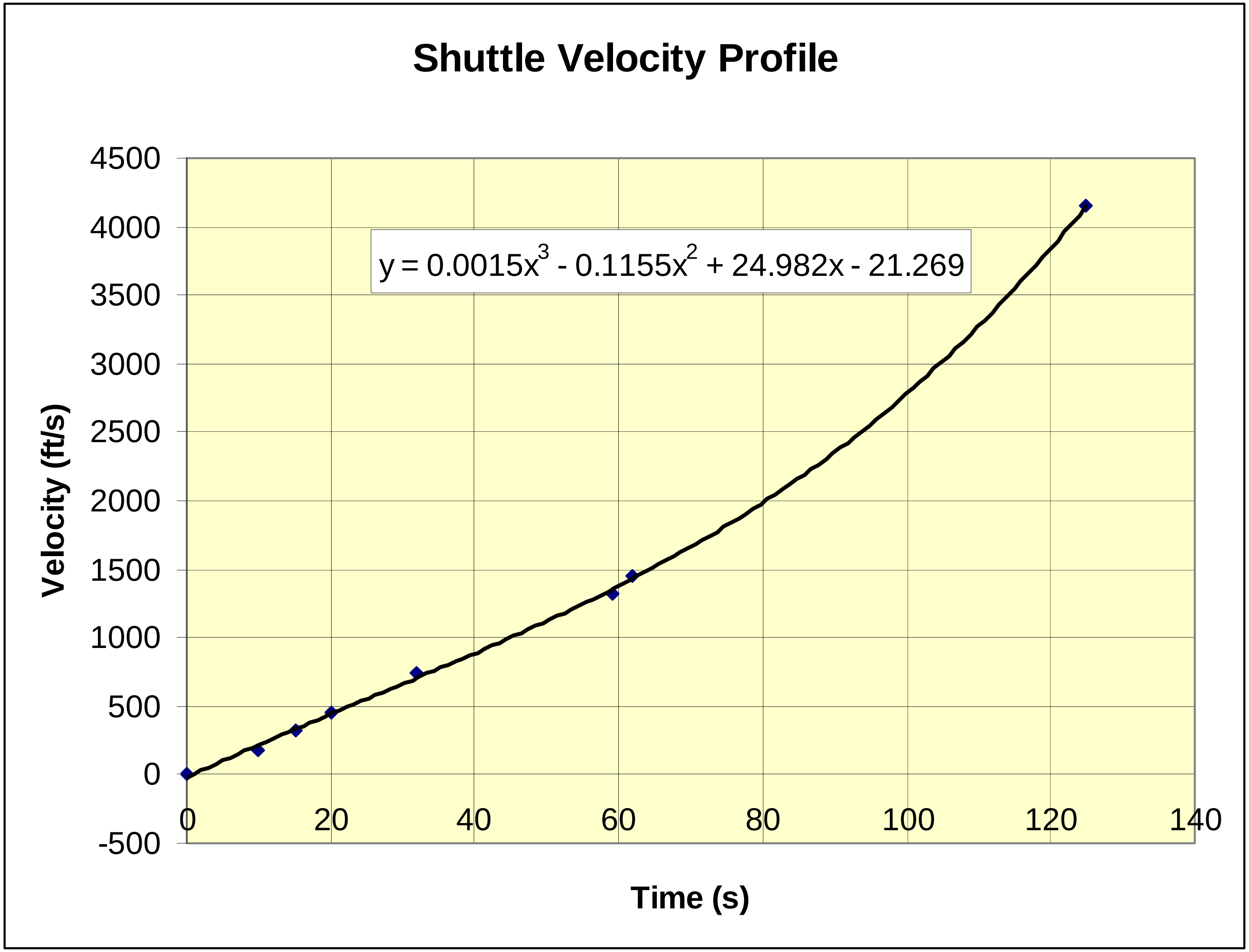
Calculus Application – Graphing and Finding Maxima or Minima

- Use a graphing calculator or computer to find the cubic polynomial that best models the velocity of the shuttle for the time interval $0 \leq t \leq 125$. Then graph this polynomial.

- Find a model for the acceleration of the shuttle and use it to estimate the maximum and minimum values of acceleration during the first 125 seconds.

Strategy!

- Let us use a computer program (MS-EXCEL) to graph the variation of velocity with time for the first 125 seconds of flight after liftoff.
- The graph is first created as a scatter plot and then a trend line is added.
- The trend line menu allows for the selection of a polynomial fit and a cubic polynomial is picked as required in the problem description above.



Shuttle Velocity Profile
y = 0.0015x3 - 0.1155x2 + 24.982x - 21.269
4500
4000
3500
3000
2500
2000
1500
1000
500
0
-500
0
20
40
60
80
100
120
140
Velocity (ft/s)
Time (s)

Solution

- From the graph, the function *y(x)* or *v(t)* can be expressed as

$$v(t) = 0.0015t^3 - 0.1155t^2 + 24.982t - 21.269$$

- Acceleration is the derivative of velocity with time.

$$a(t) = \frac{dv(t)}{dt} = 0.0045t^2 - 0.231t + 24.982$$

Solution Continued

- During the first 125 seconds of flight, that is in the interval $0 \le t \le 125$; apply the Closed Interval Method to the continuous function $a(t)$ on this interval. The derivative is

$$a'(t) = \frac{da(t)}{dt} = 0.009t - 0.231$$

- The critical number occurs when $a'(t) = 0;$

which gives us $t_1 = \dfrac{0.231}{0.009} \approx 25.67$ seconds.

Solution Continued

- Evaluating the acceleration at the Critical Number and at the Endpoints, we get

$$a(25.67) = 22.0 \ \ ft / s^2$$

$$a(0) = 24.982 \ ft / s^2$$

$$a(125) = 66.42 \ \ ft / s^2$$

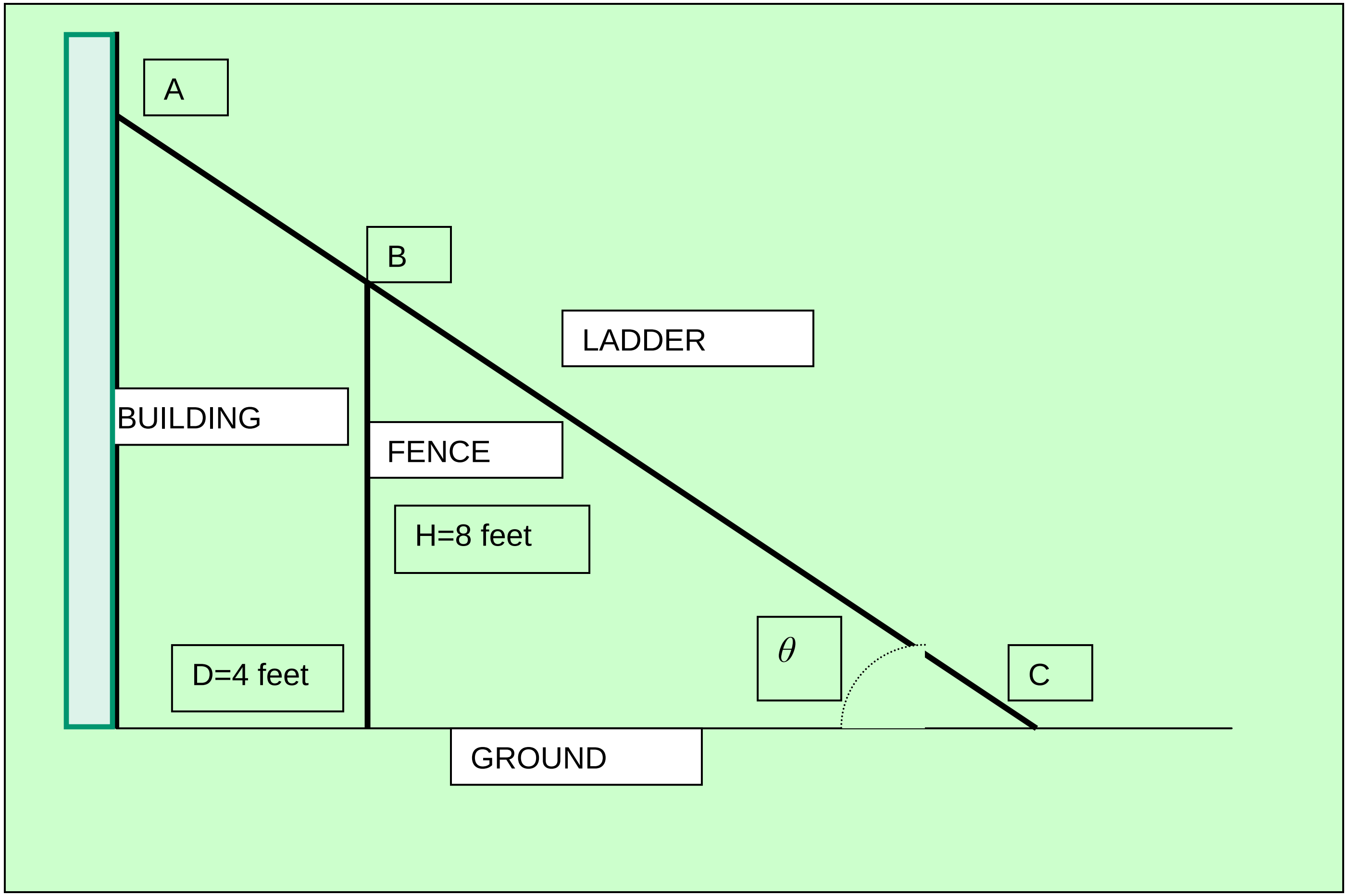
- Thus, the maximum acceleration is <u>66.42 ft/s^2</u> and the minimum is <u>22.0 ft/s^2</u>.

Calculus Application – Optimization

Section 4.7 #34:

- A fence is 8 feet tall and runs parallel to a tall building at a distance of 4 feet from the building.
- What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

Calculus Application – Optimization



Calculus Application – Strategy

- From the figure, using trigonometry, the length of the ladder can be expressed as

$$L(\theta) = \text{BC+AB} = \boxed{\frac{H}{\sin\theta} + \frac{D}{\cos\theta}}$$

- Next, find the critical number for θ for which the length L of the ladder is minimum.
- Differentiating $L(\theta)$ with respect to θ and setting it equal to zero.

Engineering Courses with Math

- Some future Engineering Courses at UCF that you may take related to this topic are
  - **EGN 3310** – Engineering Mechanics – Statics
  - **EGN 3321** – Engineering Mechanics – Dynamics
  - **EGN 3331** – Mechanics of Materials
  - **EML 3601** – Solid Mechanics
- and several of your engineering major courses
- Often, the use of derivatives is a part of our routine engineering calculations in some form

Use of Calculus in Engineering

- Real-world Engineering Applications that use Calculus Concepts such as Derivatives and Integrals
- Global and Local Extreme Values are often needed in optimization problems such as
  - Structural or Component Shape
  - Optimal Transportation Systems
  - Industrial Applications
  - Optimal Biomedical Applications

Calculus Topics Covered

- Global and local extreme values
- Critical Number
- Closed Interval Method
- Optimization Problems using Application to Engineering Problems

Applications to Engineering

- Maximum Range of a Projectile – (Mechanical and Aerospace engineering)
- Optimization of Dam location on a River (Civil engineering)
- Potential Energy and Stability of Equilibrium (Mechanical, Civil, Aerospace, Electrical Engineering)

Applications to Engineering

- Optimal Shape of an Irrigation Channel (Civil engineering)
- Overcoming Friction and other Forces to move an Object (Mechanical, Aerospace, Civil engineering)
- Beam Design (All Engineering)