

Applications of Calculus I

October 22, 2008

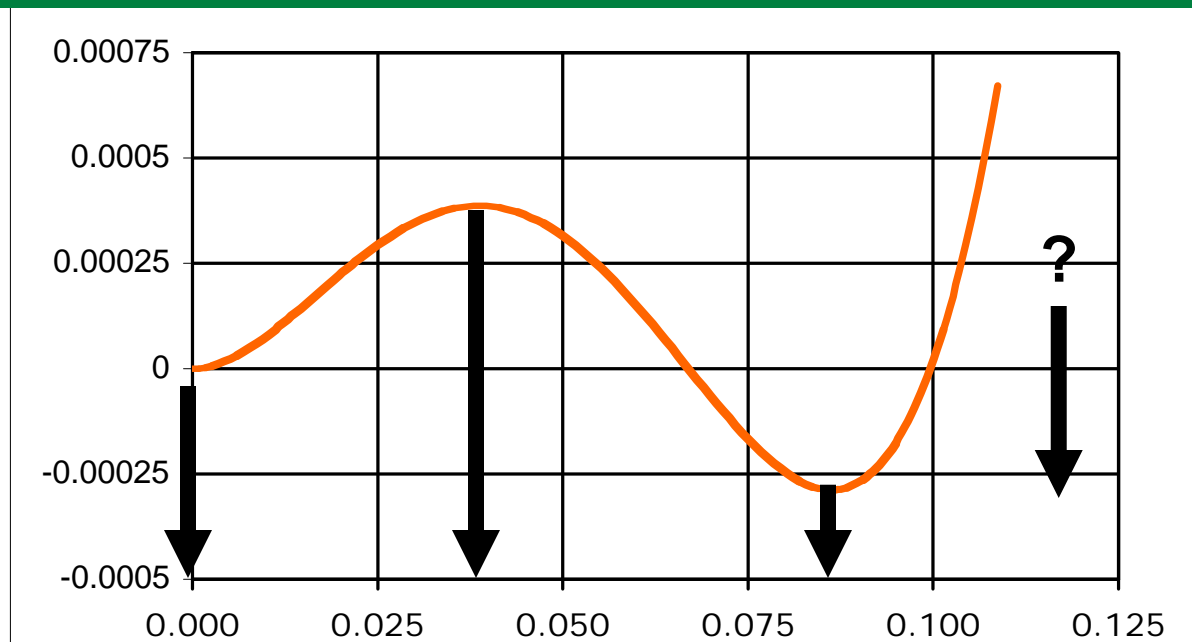
1. Methods for calculating a derivative in real life
2. Building a quantum star
3. Thinking four-dimensionally

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Department of Physics



Our goal for this afternoon: central density and redshift in a quantum star

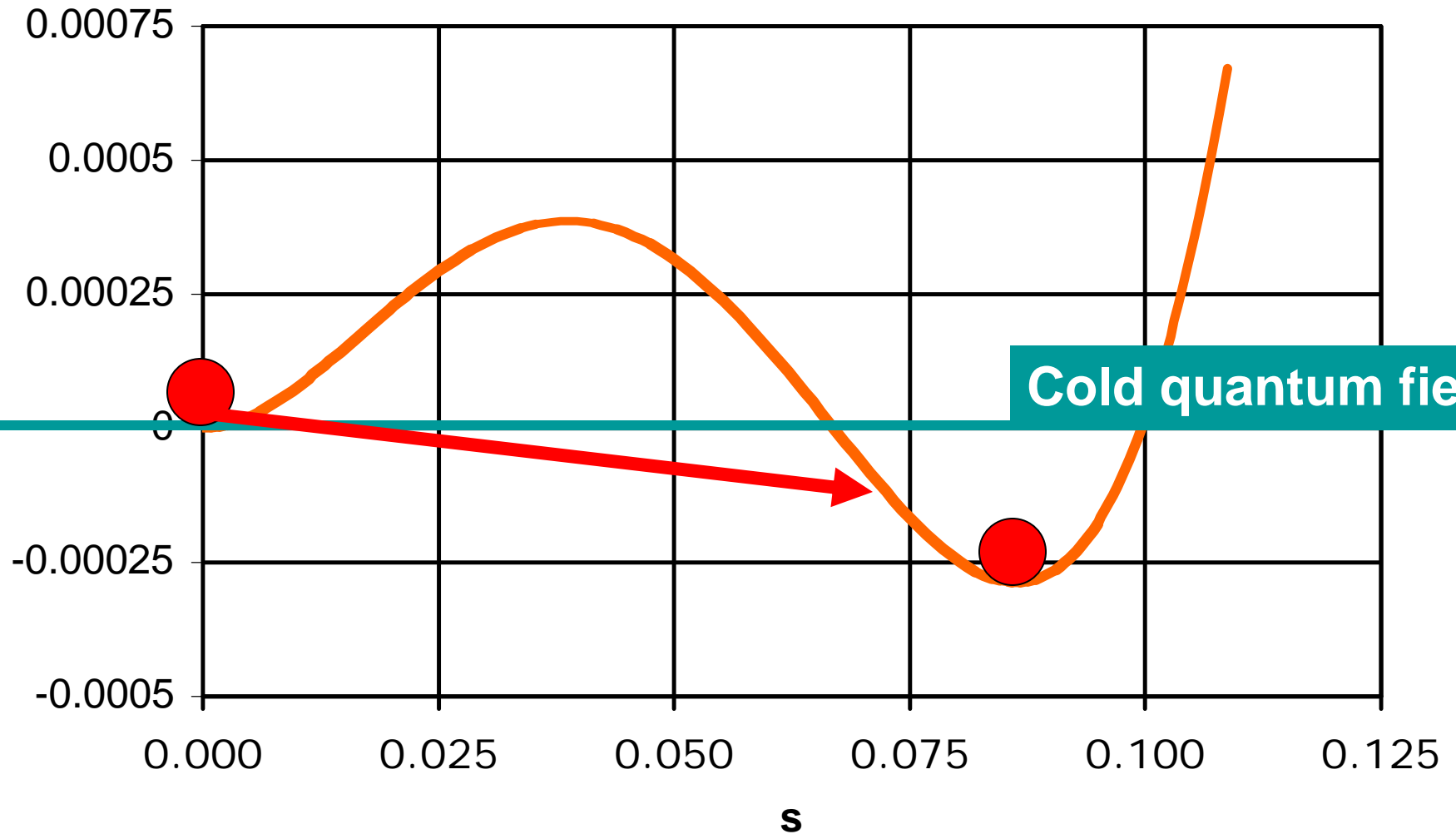
RECAP: critical points.



- A. Global minimum energy, V_{\min}
- B. Global maximum energy, V_{\max}
- C. Local minimum energy
- D. Local maximum energy
- E. And the “height” of $V(s)$ at each of these!!!

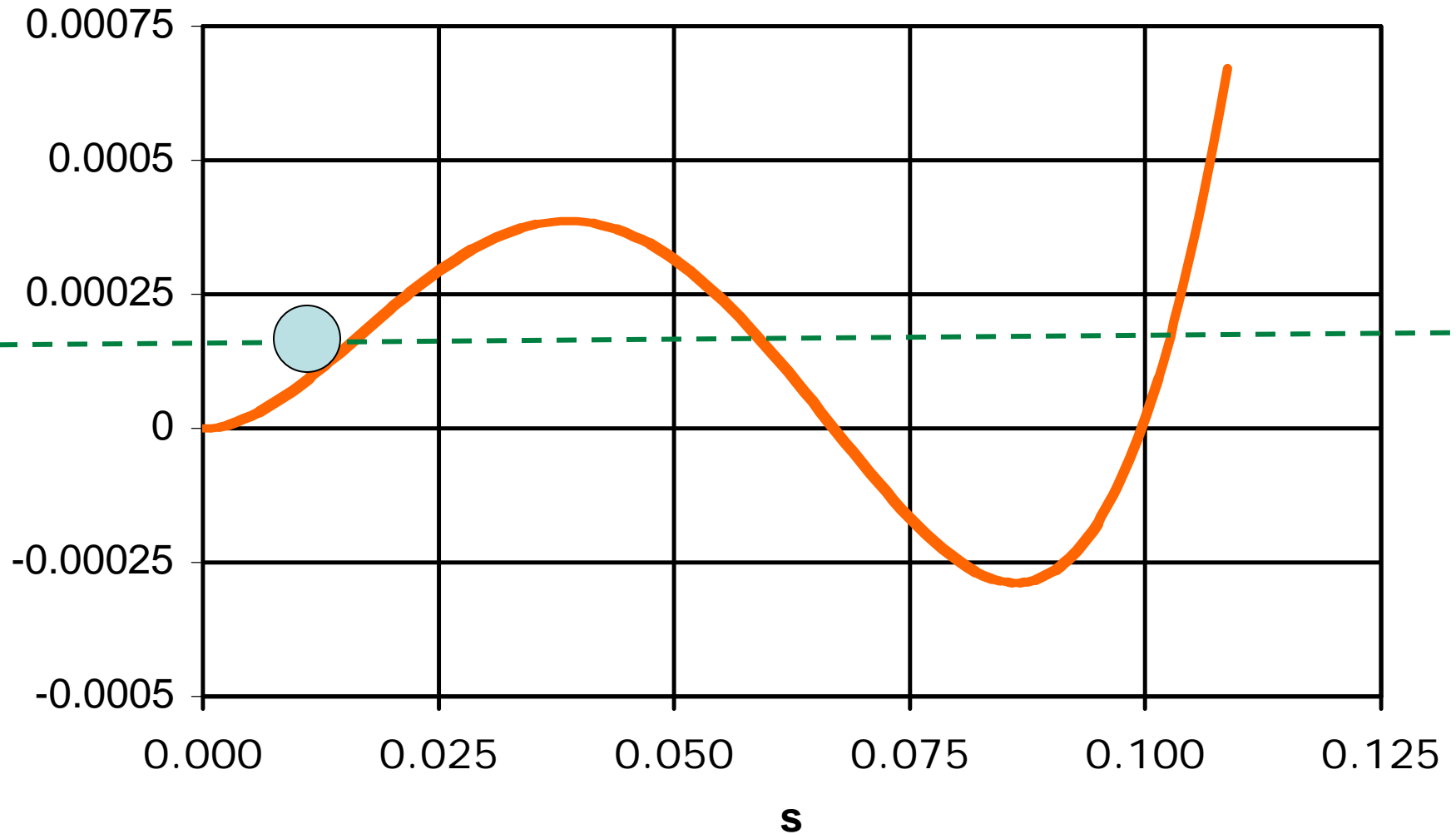
RECAP: tunneling

Quantum field energy function $V(s)$



Building a quantum star

Quantum field energy function $V(s)$



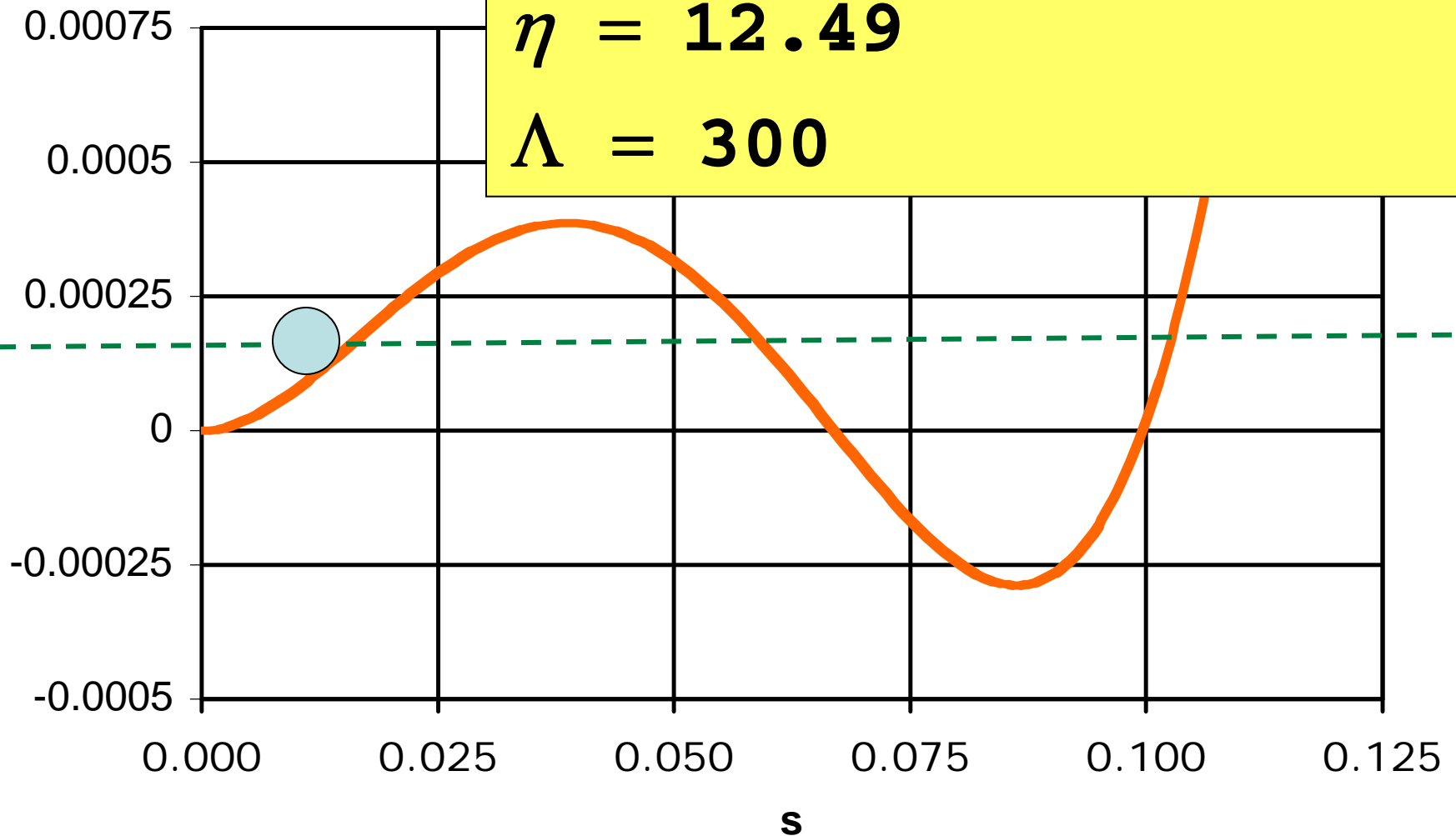
Building a quantum star

Quantum

$$V(s) = s^2 - 2\eta s^3 + \frac{1}{2} \Lambda s^4$$

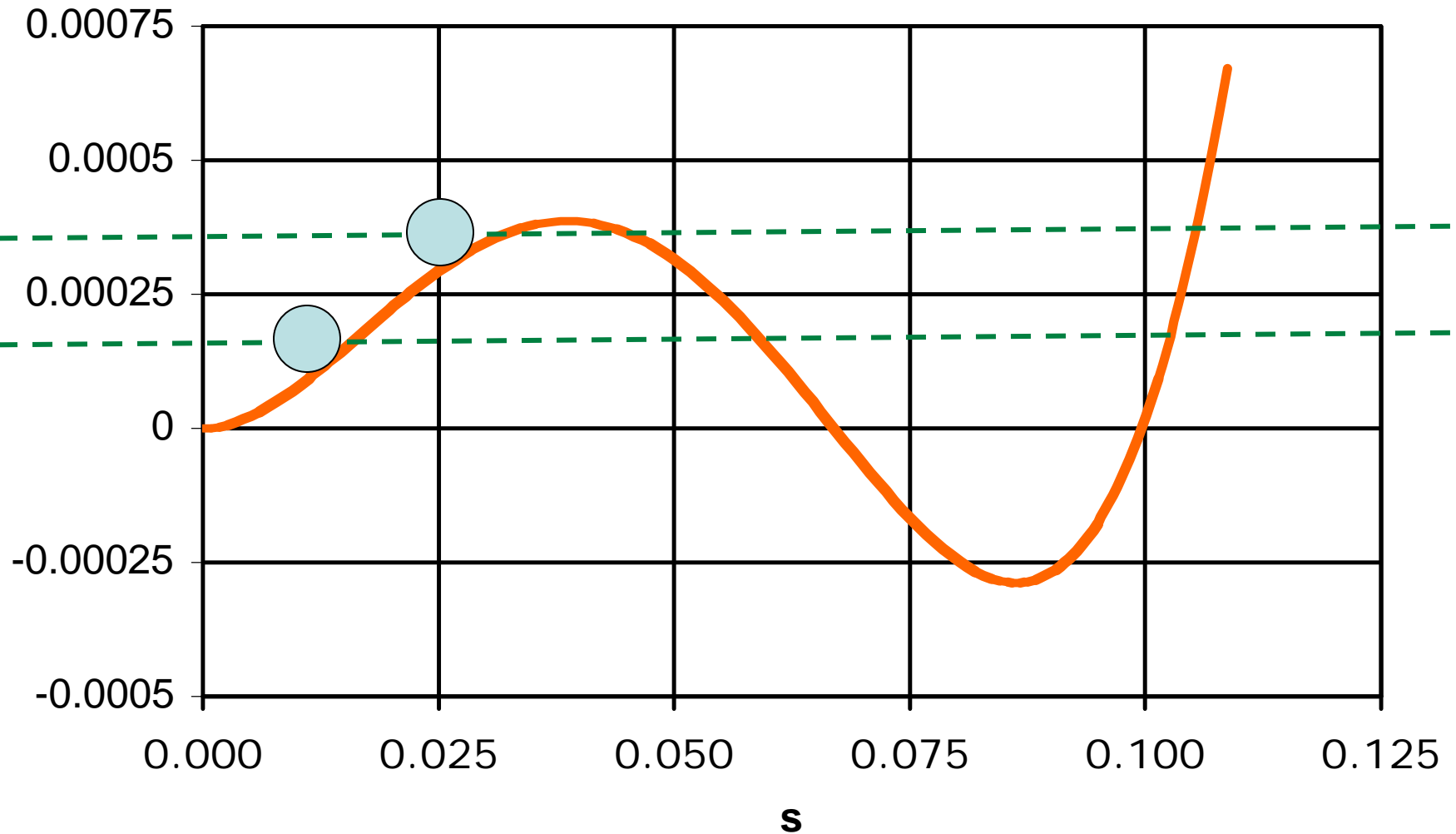
$$\eta = 12.49$$

$$\Lambda = 300$$

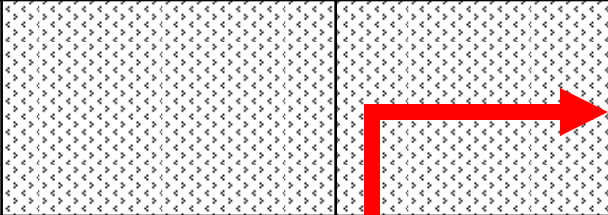
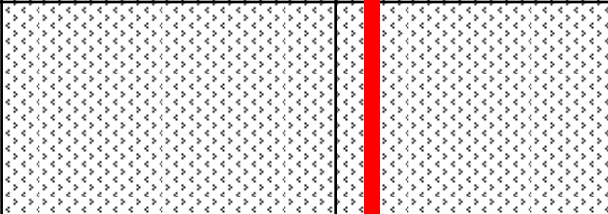
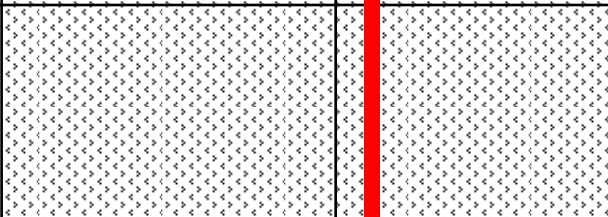


Building a quantum star: starting points

Quantum field energy function $V(s)$

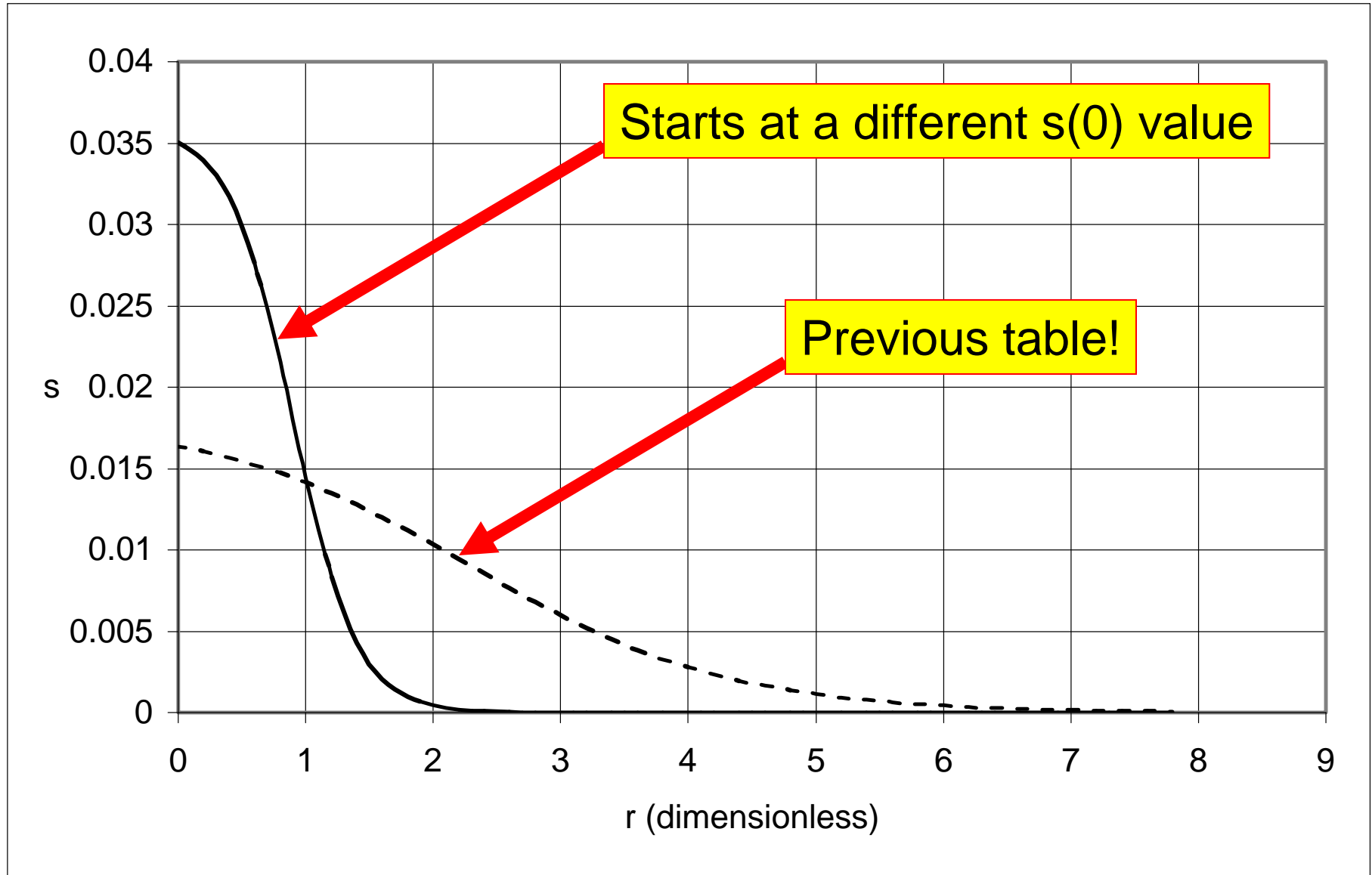


Building a quantum star: starting points

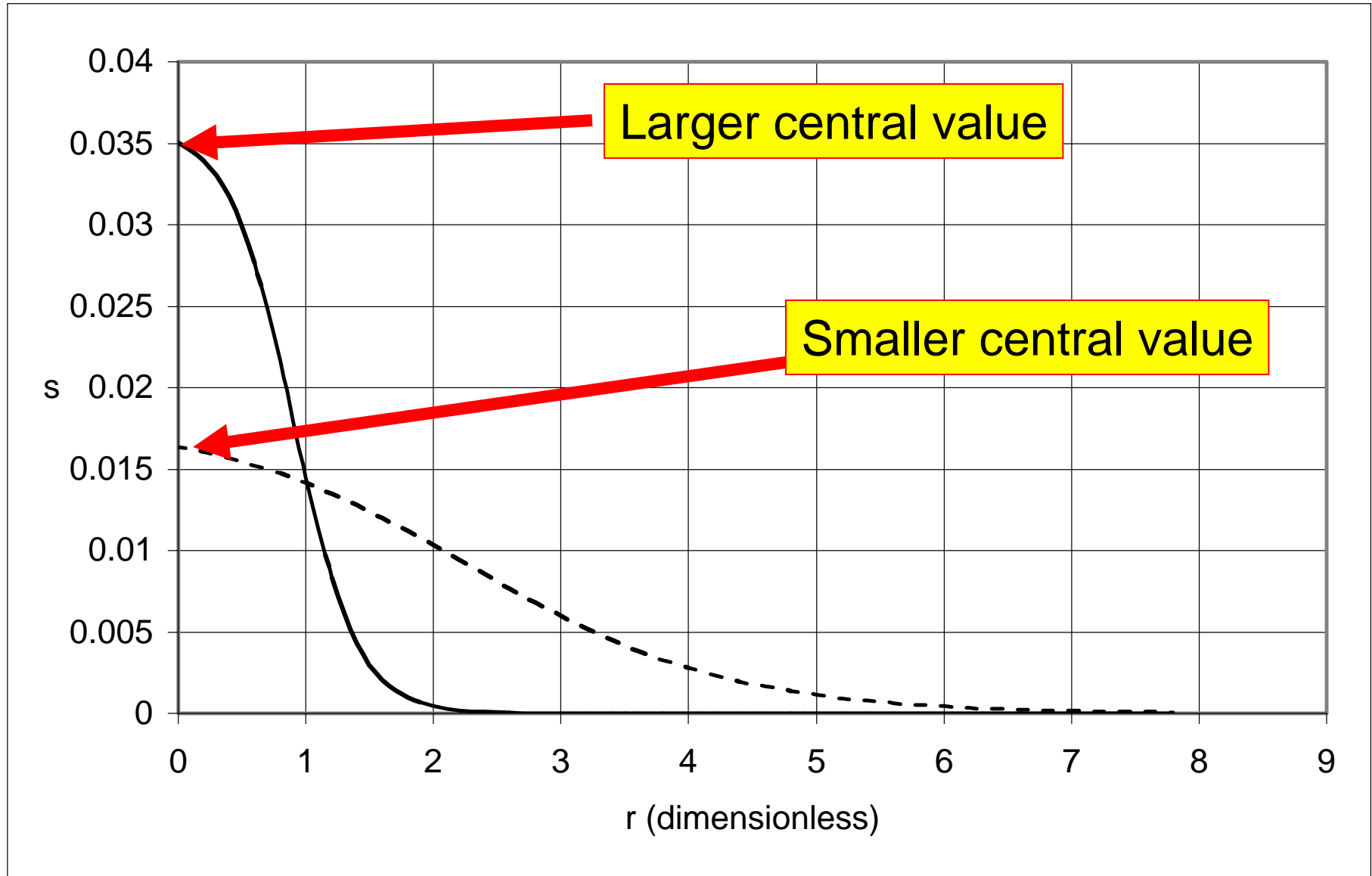
Position r	Computational columns, s' etc.	Field $s[r]$
0		0.0164 = $s(0)$
$0 + dr$		Next value of s
$0 + 2dr$		Etc.

What you start with here will determine the rest of $s[r]$.

Building a quantum star: starting points



Building a quantum star: starting points



Central density

$$\rho = \alpha [\mathbf{s}(\mathbf{0})]^2 \quad \text{E.g., grams per cm}^3$$

Strategy:

If your observational or lab work gives you reason to think that “clumps” of this field have a certain density, then it allows you to set up your table....

Maybe figure out a good prediction

Central density

Position r	<i>Computational columns, s' etc.</i>		Field $s[r]$
0			0.0164 = $s(0)$
			next value of s
			.

Strategy:

If your observational or lab work gives you reason to think that “clumps” of this field have a certain density, then it allows you to set up your table....

Central density

Position r	<i>Computational columns, s' etc.</i>		Field $s[r]$
0			0.0164 = $s(0)$
			next value of s
			.

Strategy:

... it allows you to set up your table and your table of derivatives does all the heavy lifting, to get you all other values of $s[r]$.

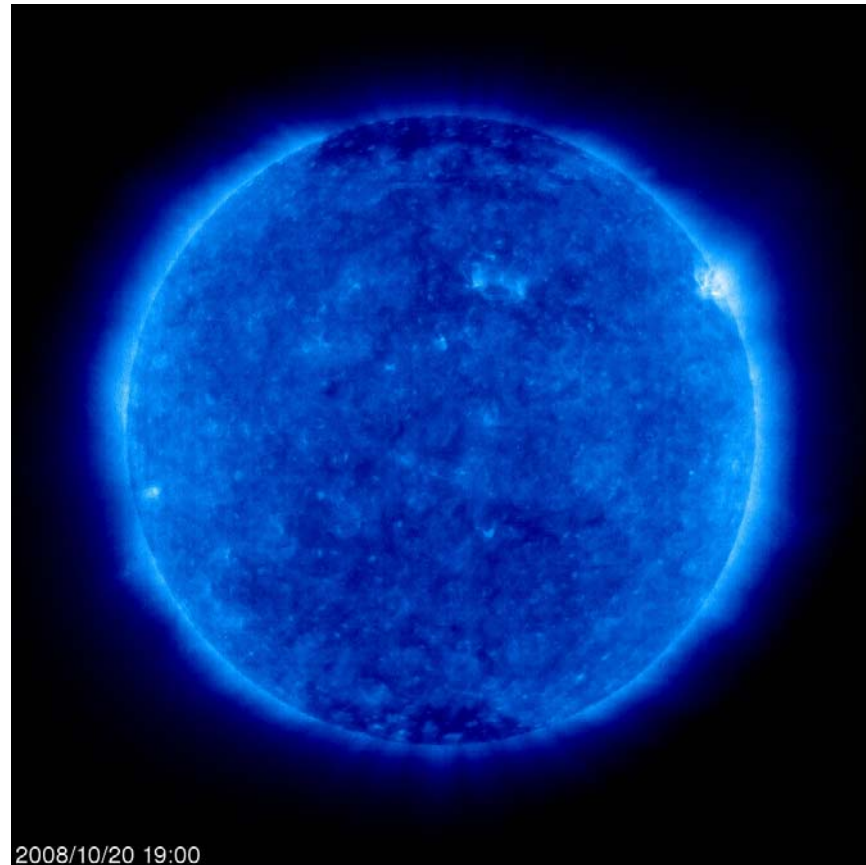
E.g., Klein-Gordon equation!

$$s'' - \frac{1}{c^2} \mathcal{E} = \left[\frac{mc}{h} \right]^2 s(r)$$

Use derivatives to get $s[r]$!
Other columns in table!!!

U want this

Central redshift means ...what?



Central redshift

- A. If one works in curved spacetime, there is central redshift to worry about.
- B. Gravity cannot change the speed of a photon...
- C. But can change its frequency
 - ✓ I.e., its color.

$$s'' - \frac{1}{c^2} \ddot{s} = \left[\frac{m c}{\hbar} \right]^2 s(\mathbf{r})$$

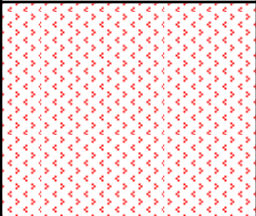
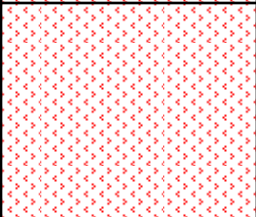
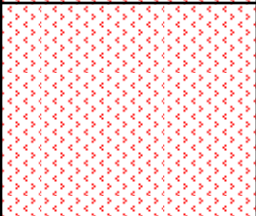
Central redshift

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$$s'' - \frac{1}{c^2} \ddot{s} = \left[\frac{mc}{\hbar} \right]^2 s(r)$$

Two new functions,
 $A[r]$ and $B[r]$

Table for curved space

Position r	<i>Comp</i>	Field $s[r]$	$A[r]$	$B[r]$
0		0.0164	1.00	0.0765
$0 + dr$		Next value		
$0 + 2dr$		Etc.		

Central redshift

1. Taking into account gravity, we will definitely see changes in the shape of $s[r]$.
2. But now you have three starting values to think about and adjust.
3. Some values of $s[0]$ make A and B drive everything into a black hole!
4. But the central redshift is the rough value of gravitational strength and...

Central redshift

5. Doppler shift of a photon of light IF it were emitted from the center of the star.
6. It is the inverse of the starting value of B !
7. $1/B(0)$
8. So...pay attention to how you handle your derivatives!!!!