Detecting Edges in Images: Day 2

by
Dr. Niels Lobo
UCF EXCEL Applications of Calculus
Images and their Edges

Computer Vision
Good for substituting machine in place of eye

- Can assist with recognition
- Can assist with navigation
- Can assist with manipulation
Computer Vision: Helpful Mirror
Computer Vision: Driverless Cars

Automated Driver Console

Battlefield

Urban Driving

Driverless Taxis
Computer Vision

Airport Security
Computer Vision

Monitor U.S. Assets Abroad  UCF EXCEL
Computer Vision

Border Security

UCF EXCEL

NSF
Medical Imaging

Image Guided Surgery

3D Models

Automated Cancer Scans

Computerized Fracture Estimation

Revolutionizing Medical Science
Computer Vision

A Basic Task: Detect Edges of Regions
Detecting Edges in an image

This is an example of a picture you might see on a computer screen.

20 X 20 pixel image of black box on square white background.

Pixel Values for image

```
255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
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255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
255 255 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 255
```
Computer Vision
Plot values from a row

-20 0 20 40 60 80 100 120

Pixel Values
Find jumps in the plot

Denote the plot by $I(x)$, then we compute

$$\frac{I(x) - I(x-1)}{x - (x-1)}$$

We can think of two values, A and B, moving along the row.

So we get the calculation being merely $(B-A)/1$
Again, the plot of a row
Plot of difference of pairs B-A
Absolute Value of B-A

Pixel Values
How to find strong edges

To find an edge from this derivative plot, use a threshold.
Effect of Thresholding

Threshold Bar

Pixel Values
Back to the other example
Back to the other example
This one has a drop and then a rise.
Difference of pairs, B-A

Pixel Values
Absolute Value of (B-A)
A Basic Task: Detect Edges of Regions
Consider a typical image
For this typical image, we know we can find the edges as we proceed along the horizontal direction, i.e., along a row, i.e., the x-direction.
For this typical image

What about the vertical direction???

i.e., along a column?  i.e., along the y-direction?
The y-direction
So, just as for the x-direction, we can compute the difference quotient for the y-direction:

\[ \frac{I(y) - I(y - 1)}{y - (y - 1)} \]

which means we are to compute the difference between two neighboring points that are vertical.
So, at all points on the image, we have 2 answers (one from x-direction and one from y-direction.) How to give a unified answer at all the points on the image?
So, denote image by \( I(x, y) \)

Then, the two Difference quotients are:

\[
\frac{\partial I}{\partial y} \quad \text{and} \quad \frac{\partial I}{\partial x}
\]
What to call these two?

So, get the notion of the Gradient.

The two quantities combine to give the Gradient Vector. Page 1095 of text.
The Gradient Vector is a physical descriptor of two dimensional functions;

The symbol for the gradient vector is \( \vec{\nabla} I \), and to repeat, it has two parts, the partial derivatives

\[
\frac{\partial I}{\partial x} \quad \text{and} \quad \frac{\partial I}{\partial y}
\]
The magnitude of the gradient vector can be obtained by squaring the individual components, adding them, and taking the square root, to get one scalar number. This concept is introduced in your Calculus textbook on page 1095, Chapter 17.
The magnitude of the gradient vector can be obtained by squaring the individual components, adding them, and taking the square root,

\[ \mathbf{\nabla} I \text{'s magn} = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \]
Apply the gradient magnitude computation to this image
Gradient Magnitude
Use a Threshold
Use a LOWER Threshold

Edges too Thick!!
These Edges are Thick

The edges are thick. Let us see how we can get thinner edges.
Let us examine the values in the data near an edge.

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</tbody>
</table>
Thickness of edges: How to fix

Plot these values

- Pixel Values

0 20 40 60 80 100 120 140
11 11 18 95 116 129 132

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Plot the derivative (the diff quotient)

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</tr>
</thead>
<tbody>
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<td>$I'$</td>
<td>--</td>
<td>-1</td>
<td>1</td>
<td>0</td>
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<td>35</td>
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<td>6</td>
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<td>3</td>
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</tr>
</tbody>
</table>
Plot the derivative (the diff quotient)
Threshold this plot

Pixel Values

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Plot the derivative (the diff quotient).

So, there are several nearby points that are above threshold.

This leads to the thick edges.

Need to be thinner.

Find the peaks of the curves.
Plot the derivative (the diff quotient).

So, there are several nearby points that are above threshold.

This leads to the thick edges.

Need to be thinner.

Find the peaks of the curves.
Finding Peaks in First derivative

With any curve, a flat section can be found by taking its derivative, and identifying where the derivative is zero.

We want a flat section that is a peak (or trough) in the first derivative.
Finding Peaks of Derivative plot

Take another derivative of the plot. (Section 3.8)

\[ I' = \frac{\partial I}{\partial x} = \frac{I(x) - I(x - 1)}{x - (x - 1)} \]

\[ I'' = \frac{\partial^2 I}{\partial x^2} = \frac{I'(x) - I'(x - 1)}{x - (x - 1)} \]
To find peaks, take second derivative

\[ I'' = \frac{\partial^2 I}{\partial x^2} = \frac{I'(x) - I'(x - 1)}{x - (x - 1)} \]

<table>
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<td>28</td>
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<td>-2</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>
To find the peaks

\[ I'' = \frac{\partial^2 I}{\partial x^2} = \frac{I'(x) - I'(x - 1)}{x - (x - 1)} \]
To Find Peaks of First Derivative plot

So, just find places where values progress from positive to negative, and vice-versa.

Need to make sure that the jump from positive to negative (or vice versa) is large enough.
Finding Peaks of Derivative plot

We know

\[ I' = \frac{\partial I}{\partial x} = \frac{I(x) - I(x - 1)}{x - (x - 1)} \]

\[ I'' = \frac{\partial^2 I}{\partial x^2} = \frac{I'(x) - I'(x - 1)}{x - (x - 1)} \]
What about Vertical direction?

Similarly,

\[
\frac{\partial I}{\partial y} = \frac{I(y) - I(y - 1)}{y - (y - 1)}
\]

\[
\frac{\partial^2 I}{\partial y^2} = \frac{I'(y) - I'(y - 1)}{y - (y - 1)}
\]
Peaks

Combining the two values, gives the Laplacian

Which is defined as

\[
\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}
\]

And is given the symbol \( \nabla^2 \)

Note that this is not a vector.
Computing $\nabla^2$, the Laplacian:

So, given $I(x, y)$

Can compute

$$\frac{\partial^2 I}{\partial y^2} \quad \text{and} \quad \frac{\partial^2 I}{\partial x^2}$$

Then add them up.
The Laplacian of the Picture
To Find Peaks of Derivative plot

So, just find places where values progress from positive to negative, and vice-versa.

Mark the centers of these changing patterns.
Mark these positions of change
Review of Laplacian

Combine the two partial derivative values, to get the Laplacian

Which is defined as

\[
\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}
\]

Note that this is not a vector.
After Find Edges, Lay them Straight

To Straighten out edges, can use the calculus topic of Optimization (Section 4.7).

This generally involves trying to find a choice from among many alternatives. The choice is one that minimizes (or maximizes) some Function. So, it is common to solve this by taking the Derivative of the Function, setting it to zero and determining where the derivative is zero.
Straightening is best line thru’ region
Best line is measured by distances
Now, we can add up the total of all distances

$$\chi^2 = \sum_i (dist_{pi})^2$$ (1)
Find line that minimizes total distances

Our task then is to find the line that minimizes \( \chi^2 \)
Straighten Curves

We do not delve into details of the Straightening-out process here.
Some Arty stuff
Computer Vision
Computer Vision
Can use Derivative in time, as well
Can use Derivative in time

This is good for video data (movies).

So, we can take derivatives of $I(x, y, t)$.

If we hold $x, y$ constant, we are fixing a pixel, and asking how it changes in time.

Compute $\frac{\partial I}{\partial t}$ at all fixed $x, y$. 
How to use Derivative in time?

The derivative in time tells us where in the image, the pixels are changing, and what those changes are. Zero means no change at those pixels.

It is the same as subtracting one frame of the movie from another.
Vehicle Guard: An Example
Vehicle Guard: Example Results

Key

- Object is near vehicle
- Object touching vehicle
- Reach-in on left side
- Reach-in on right side

Note: Purple spots are where vehicle is touched

results.avi
Detecting Person-On-Person Violence

People Hitting Each Other
Detecting Person-On-Person Violence

Some examples
Detecting Person-On-Person Violence

Need to compute *Jerk* of limbs and head

This involves third derivative in time

Section 3.8
Use Two different methods

One uses Calculus, other uses Statistics
Detecting Person-On-Person Violence

People Hitting Each Other
Counting Heads (not just faces)
Face Orientations
Face orientations

Uses a method from Matrix Algebra
Detector Results