Application of Limits in Heat Transfer
- heat conduction

By

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UCF EXCEL Applications of Calculus
Part I – background and review of limits
(Wednesday 3 September, 2008)

1. background and motivation
2. limits: Chapter 2, sections 2.2-2.5, of J. Stewart, Calculus.
3. rates of change: Chapter 2, section 2.6 of J. Stewart, Calculus.

Part II – applications (Wednesday 10 September, 2008)

1. Review of limits and rates of change with applications to the temperature in the wall of a house and in a cooling fin
3. Determining the heat flux
4. Why you can’t take the limit on a computer
Review of limits and rate of change

1. Background:
   - In our previous discussion we covered:
     1. limits:
        \[ \lim_{x \to x_0} f(x) \]

     2. rate of change of \( f(x) \):
        \[ m_f(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

   - Calculus (Isaac Newton and Gottfried Leibniz) is principally concerned with:
     1. Differentiation: the instantaneous rate of change (slope) of the curve describing a function.

     2. Integration: the area under that curve.
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Review of limits and rate of change (cont’d)

- The notion of the limit is central to understanding the two basic operations in calculus: the derivative and the integral.

**Key Idea #1:** What we mean by **taking a limit of a function**, \( \lim_{x \to x_0} f(x) \), is to **evaluate the behavior of the function,** \( f \), as the variable on which it depends (called the independent variable), \( x \), **tends towards a particular value,** \( x_0 \).

- Often the value of the variable is a finite constant, however, in many cases we are interested in what happens when the constant is zero or value of the constant tends to negative or positive infinity.

\[
\lim_{x \to 0} f(x) \quad \lim_{x \to -\infty} f(x) \quad \lim_{x \to +\infty} f(x)
\]
Review of limits and rate of change (cont’d)

- **Basics of notation:** dependent variable, independent variable, constants and all that stuff

- **What does \( f(x) \) mean?**

\[
f(x) = 95 - 20x
\]

- \( f \) is the dependent variable (it depends on \( x \) which appears in parenthesis)
- \( x \) is the independent variable on which \( f \) depends, and it takes on all the values in its assigned/designated range, e.g. \( x \in [0,1] \)
Now we take it to the next step:

1. define two constants:
   
   $C_1 = 95$
   
   $C_2 = -20$

2. write the same function, $f(x) = 95 - 20x$, using the constants we defined above:

   $$f(x) = C_1 + C_2x$$

- **Question:** How many variables in this equation?
- **Answer:** **one** independent variable: $x$
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Review of limits and rate of change: basics of notation (cont’d)

- Now let’s put this function in the context of an application, and associate units with the variables

\[ f(x) \rightarrow T(x) \]

- Temperature \([\circ C, K, \text{ or } \circ F]\)
- \(x\) – spatial position \([m \text{ or } ft]\)

- Recall clicker question 2:

\[ T_h = 95\circ F (35\circ C) \]
\[ T_c = 75\circ F (23.9\circ C) \]
\[ L = 1\text{ ft} (0.3048 \text{ m}) \]
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Review of limits and rate of change: basics of notation (cont’d)

- The temperature in the wall is:
  \[ T(x) = T_h + (T_c - T_h) \left( \frac{x}{L} \right) \]
  \[ T_h = 95^\circ F \]
  \[ T_c = 75^\circ F \]
  \[ L = 1 \text{ ft} \]
  \[ x \text{ has units of } [\text{ft}] \text{ and } x \in [0,1 \text{ ft}] \]

- Let’s define two constants:
  \[ C_1 = T_h \quad \text{or} \quad C_1 = 95 \text{ [^\circ F]} \]
  \[ C_2 = \frac{(T_c - T_h)}{L} \quad \text{or} \quad C_2 = -20 \text{ [^\circ F/ft]} \]

- Let’s write the temperature in the wall using these two constants:
  \[ T(x) = C_1 + C_2 \cdot x = 95 \text{ [^\circ F]} - 20 \text{ [^\circ F/ft]} \cdot x [\text{ft}] \]
Question: does this temperature distribution make sense?

Answer: check temperature solution in the limits of both ends of the wall

1. check the limit as $x \to 0$:
\[
\lim_{x \to 0} \left[ T_h + (T_c - T_h) \left( \frac{x}{L} \right) \right] = T_h
\]

2. check the limit as $x \to L$:
\[
\lim_{x \to L} \left[ T_h + (T_c - T_h) \left( \frac{x}{L} \right) \right] = T_c
\]

… and since the solution satisfies the imposed temperature on both ends of the wall, and the Temperature goes from high to low, the solution makes sense so far.
**Key Idea #2:** What we mean by the instantaneous rate of change of a function, say $f(x)$, with respect to its independent variable, $x$ in this case, is to evaluate the special limit defined by

$$m_f(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

- **taking this limit produces the instantaneous slope of $f(x)$ at any $x$.**
Review of limits and rate of change (cont’d)

The instantaneous rate of change of the temperature in the wall is:

\[
m_T(x) = \lim_{h \to 0} \frac{T(x + h) - T(x)}{h} = \lim_{h \to 0} \frac{\left[ T_h + (T_c - T_h) \left( \frac{x+h}{L} \right) \right] - \left[ T_h + (T_c - T_h) \left( \frac{x}{L} \right) \right]}{h} = \lim_{h \to 0} \frac{(T_c - T_h) \left( \frac{h}{L} \right)}{h} = \frac{(T_c - T_h)}{L}
\]

or \( m_T(x) = -20 \, ^\circ F/ft = -36 \, ^\circ C/m = -36 \, [K/m] \)

**Question:** does this result make sense?  
**Answer:** yes, the slope of a linear function is constant
The heat transferred per unit area of the wall, \( q \, [W/m^2] \), is given by the relationship discovered by Jean-Baptiste Fourier (1768-1830):

\[
q = -k \times m_T(x)
\]

- \( k \) is a physical property of the material, and it is called the thermal conductivity. It is measured and known for many materials.
- and using our temperatures and instantaneous rate of change of the temperature with respect to space, we obtain for our case of a 1ft thick wall of concrete, a heat loss of:

\[
q = -(1.1 \, [W/\text{m}K]) \times (-36 \, [\text{K/m}]) = 40 \, [W/m^2]
\]

or 40 Watts per square meter of wall comes into the house and has to be removed by AC.
Limits and rate of change: heat transfer in a cooling fin

- determine the temperature distribution in a cooling fin.
- Fins are designed to help improve the removal of heat and are ubiquitous in our world: car engine cooling system where fins are attached to the car radiator, computers where fins are attached to certain electronic chips to aid in removing waste heat.
- A fin of length, $L$, is being cooled by air at temperature $T_c$ that is being forced over the fin by a fan.
The general solution for the temperature is:

\[ T(x) = T_c + C_1 e^{-\lambda x} + C_2 e^{+\lambda x} \]

where \( C_1 \) and \( C_2 \) are arbitrary constants, and \( \lambda \) is a constant whose value depends on the fin geometry and fin material property as well as how fast the air is blowing over the fin with units \([m^{-1}]\).

**Question:** what is the dependent variable for the temperature equation?  
**Answer:** \( x \)

**Question:** what are \( T_c, C_1, C_2, \) and \( \lambda \) in this equation?  
**Answer:** constants

Thus, using the limiting behavior of the exponential function as \( x \to \infty \), we arrive at the conclusion that the general solution to the very long fin problem is

\[ T(x) = T_c + C_1 e^{-\lambda x} \]

The remaining solution tells us that as the fin becomes very long, the temperature tends to that of the cooling air, \( T_c \), which makes physical sense.
The temperature distribution in a very long cooling fin is then

\[ T(x) = T_c + (T_w - T_c) e^{-\lambda x} \]

Plotted for a hot wall temperature of \( T_w = 150 ^\circ C \), a cooling air temperature of \( T_c = 25 ^\circ \), and taking a characteristic value of \( \lambda = 13.6 \text{ m}^{-1} \), we see the solution satisfies the two limits, namely:

\[ \lim_{x \to \infty} T(x) = T_c \quad \text{and} \quad \lim_{x \to 0} T(x) = T_w \]

We will use these results in our next discussion
Limits and rate of change: temperature in a cooling fin (cont’d)

- To find the heat removed by the fin, $Q \,[W]$, we multiply the heat flow rate per unit area, $q \,[W/m^2]$, by the fin cross-sectional area, $A \,[m^2]$, and

$$Q(x) = -kA \times m_T(x)$$

- The thermal conductivity for the fin we considered above was that of Copper, and its value is: $k = 385 \,[W/m°C]$. We also considered a cross-sectional area, $A = 2 \times 10^{-4} \,[m^2]$ and a value of $\lambda = 4.414 \,[m^{-1}]$.

- Utilizing equation 7, rules 3 and 7 of limits laws from section 2, and the law of exponents, we have now set out to find, $m_T(x)$,

$$m_T(x) = \lim_{h \to 0} \left[ \frac{T(x + h) - T(x)}{h} \right]$$
we are now required to evaluate the limit in the braces to determine \( m_T(x) \) for our problem and ultimately our heat flow rate, \( Q(x) \).
Limits and rate of change: temperature in a cooling fin (cont’d)

- A numerical computation using MATHCAD where we drive \( h \) ever smaller (we’ll talk about this later)

\[
\text{Function}(h) := \frac{e^{-\lambda h} - 1}{h}
\]

<table>
<thead>
<tr>
<th>( h_i )</th>
<th>Function ( (h_i) )</th>
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<tr>
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</tr>
</tbody>
</table>

- Seems to indicate that the limit is \(-\lambda\) since we set the value of \( \lambda = 4.414 \) in our problem.
Limits and rate of change: temperature in a cooling fin (cont’d)

- Using the symbolic manipulator in MATHCAD (actually the MAPLE symbolic manipulator engine), we find indeed that

\[
\lim_{h \to 0} \frac{e^{-\lambda \cdot h} - 1}{h} \to -\lambda
\]

- Leads us to the general result that for the function

\[
f(x) = e^{-\lambda x}
\]

- the instantaneous rate of change of \( f(x) \) with respect to \( x \) is

\[
m_f(x) = \lim_{h \to 0} \left[ \frac{e^{-\lambda (x+h)} - e^{-\lambda x}}{h} \right] = -\lambda e^{-\lambda x}
\]
Moreover, setting $\lambda = -1$, we find the extraordinary result that, the instantaneous rate of change of the function

\[ f(x) = e^x \]

is itself!!!

This amazing result states that the instantaneous rate of change of the exponential function, $e^x$, as a function of $x$ is the value of the exponential function at that location, $x$.

The exponential function is the **only function** that exhibits this property!
Interestingly along the way, we also established another result, namely, that by taking, \( \lambda = -1 \), we have

\[
\lim_{h \to 0} \frac{e^{-\lambda \cdot h} - 1}{h} \to -\lambda
\]

Computing the limit numerically:

\[
F(h) := \left( \frac{e^h - 1}{h} \right)
\]

Evaluating the limit symbolically with MATHCAD's symbolic manipulator:

\[
\lim_{h \to 0} \frac{e^h - 1}{h} = 1
\]
Limits and rate of change: temperature in a cooling fin (cont’d)

- So that we are now in a position to evaluate the heat flow rate due to the fin, and utilizing our results we find that
  \[ Q(x) = -kA \times m_T(x) \]

- with the temperature distribution:
  \[ T(x) = T_c + C_1 e^{-\lambda x} \quad \text{where } C_1 = T_w - T_c \]

- with the result that:
  \[ m_T(x) = -C_1 \lambda e^{-\lambda x} \]

- then for our fin, the heat flow rate is:
  \[ Q(x) = +kA\lambda C_1 e^{-\lambda x} \]
Application of Limits in Heat Transfer: heat conduction

Limits and rate of change: why you can’t quite take the limit on the computer

- Note that in computing we never went beyond say $h = 10^{-10}$ in evaluating our limits.

- Let’s see what actually happens when we see what happens when we try to compute the heat flow rate at $x=0$ and take $h$ to be increasing very small, and beyond $10^{-10}$ in an attempt to reach the limit of zero.

- Let’s apply this to computing our heat flow extracted by the fin:

$$Q(0) = \lim_{h \to 0} \left[ -kA \left( \frac{T(x_0+h)-T(x_0)}{h} \right) \right]_{x_0=0}$$
Application of Limits in Heat Transfer: heat conduction

**Background (cont’d)**

<table>
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<th>$Q_{\text{computed}}(h_i)$</th>
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![Graph](image-url)
Application of Limits in Heat Transfer: heat conduction

Background (cont’d)

\[
\text{error}_Q(h) := \left| Q(0) - Q_{\text{computed}}(h) \right|
\]

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<th>( \text{error}_Q(h_i) )</th>
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<tr>
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<td>( 1 \cdot 10^{-20} )</td>
<td>42.48161</td>
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</table>

Machine precision: the smallest value of \( h \) below which the computer does not recognize the difference between \( x_0 + h \) and \( x_0 \).
What happened is that the computer makes small errors (round off) every time that it

1. stores a real number (for most of the reals)
2. it computes a floating point operation (+, -, x and /)

This is due to the fact that the computer operates using the binary system (discovered by Leibniz, the co-inventor of modern Calculus).

1. bits=[0,1] to represent numbers and all instructions
2. the computer using a finite number of bits to store such numbers

Example: \( (0.1)_{10} = (0.000110011001101...)_2 \) irrational in binary
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Limits and rate of change: why you can’t quite take the limit on the computer (cont’d)

- As we take ever smaller $h=0.1$, 0.01, 0.001… then we are approaching the value of the limit $h=0$ and the error decreases at first up until around $10^{-6}$. In this range the round-off errors are not significant and the overall error decreases.

- As we decrease $h$ further then we begin to amplify the round-off errors and the overall error begins to increase in the range $10^{-6} – 10^{-16}$.

$\varepsilon_{\text{round-off}}$ due to:
1. storing $x_o$ and $h$
2. adding $x_o$ and $h$
3. evaluating $T(x_o)$
4. evaluating $T(x_o+h)$
5. subtracting $T(x_o)$ from $T(x_o+h)$

\[
\frac{T(x_o + h) - T(x_o)}{h} + \frac{\varepsilon_{\text{round-off}}}{h}
\]

as $h$ gets smaller, the round off error is amplified by $h$
Application of Limits in Heat Transfer: heat conduction

- This happens in any calculation on the computer and that is why you should learn how to take limits and rates of change analytically.

The natural number "e" defined as a limit: 
\[ e = \lim_{\varepsilon \to 0} \left(1 + \frac{1}{\varepsilon}\right)^{\varepsilon} \]

The natural number "e" defined as a limit: 
\[ e = \lim_{\varepsilon \to \infty} \left(1 + \frac{1}{\varepsilon}\right)^{\varepsilon} \]

| i  | \(\Delta x_i\) | \(f(\Delta x_i)\) | \(|f(\Delta x_i) - e|\) | \(|g(\Delta x_i) - e|\) |
|----|----------------|------------------|----------------|----------------|
| 1  | 0.1            | 2.594            | 0.125          | 0.125          |
| 2  | 0.01           | 2.705            | 0.013          | 0.013          |
| 3  | 1 \times 10^{-3}| 2.717            | 1.358 \times 10^{-3} | 1.358 \times 10^{-3} |
| 4  | 1 \times 10^{-4}| 2.718            | 1.359 \times 10^{-4} | 1.359 \times 10^{-4} |
| 5  | 1 \times 10^{-5}| 2.718            | 1.359 \times 10^{-5} | 1.359 \times 10^{-5} |
| 6  | 1 \times 10^{-6}| 2.718            | 1.359 \times 10^{-6} | 1.359 \times 10^{-6} |
| 7  | 1 \times 10^{-7}| 2.718            | 1.343 \times 10^{-7} | 1.343 \times 10^{-7} |
| 8  | 1 \times 10^{-8}| 2.718            | 3.011 \times 10^{-8} | 3.011 \times 10^{-8} |
| 9  | 1 \times 10^{-9}| 2.718            | 2.236 \times 10^{-7} | 2.236 \times 10^{-7} |
| 10 | 1 \times 10^{-10}| 2.718          | 2.248 \times 10^{-7} | 2.248 \times 10^{-7} |
| 11 | 1 \times 10^{-11}| 2.718          | 2.249 \times 10^{-7} | 2.249 \times 10^{-7} |
| 12 | 1 \times 10^{-12}| 2.719          | 2.417 \times 10^{-4} | 2.417 \times 10^{-4} |
| 13 | 1 \times 10^{-13}| 2.716          | 2.172 \times 10^{-3} | 2.172 \times 10^{-3} |
| 14 | 1 \times 10^{-14}| 2.716          | 2.172 \times 10^{-3} | 2.172 \times 10^{-3} |
| 15 | 1 \times 10^{-15}| 3.035          | 0.317           | 0.317           |
| 16 | 0              | 1                | 1.718           | 1.718           |
| 17 | 0              | 1                | 1.718           | 1.718           |
| 18 | 0              | 1                | 1.718           | 1.718           |
| 19 | 0              | 1                | 1.718           | 1.718           |
| 20 | 0              | 1                | 1.718           | 1.718           |
The instantaneous rate of change of \( f(x) \) with respect to \( x \) has a special name, and it is called the derivative of \( f(x) \).

Leibniz defined a special notation for this and here goes the definition:

\[
\frac{dy}{dx} = m_f(x_0) = \lim_{h \to 0} \left[ \frac{y(x_0 + h) - y(x_0)}{h} \right] = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}
\]

You will learn many rules to carry out this definition and these rules as based on the notion of limits.

Example:

\[
\frac{d(e^x)}{dx} = e^x
\]
Application of Limits in Heat Transfer: heat conduction

Introductory Dialog: (spoken by the "Professor", played by Lewis M. Branscomb)

To cover that great theory* would be my fondest hope,
Were it not that of this course it’s far beyond the scope,
This brings us to the question, though,
Of how much math you ought to know:
Most of it is inconsequential,
But the derivative is essential.
But you oughtn’t have trouble, ought you?
If you remember that song I taught you:

*i.e. Relativity, referring to the previous song

The Derivative Song:

You take a function of x and you call it y,
Take any x-nought that you care to try,
Make a little change and call it delta-x,
The corresponding change in y is what you find nex’,
And then you take the quotient, and now carefully
Send delta-x to zero and I think you’ll see,
That what the limit gives us, if our work all checks,
Is what we call dy/dx, it’s just dy/dx.

These lyrics were published in American Mathematical Monthly, vol.81, p. 490 (1974).
Applications to computational fluid dynamics and heat transfer:

Limits, rates of change and derivatives are used to build numerical models to solve problems via Finite difference methods and the closely related finite volume method are used widely to solve problems in a variety of fields:

- Power generation
- Aerospace
- Defense
- Heating, ventilations and refrigeration
- Oil and Gas industry
- Semiconductor
- Polymer processing
- Biomedical engineering
- Nuclear
- Marine and coastal engineering
- …
**Conservation principle:** our basic tool

\[ [\text{IN}] - [\text{OUT}] + [\text{GEN}] = \text{net time rate of change} \]

\[ [\ ] = \text{mass, linear momentum, energy, species, angular momentum, ...} \]

money in the bank
Conclusions

- Calculus is concerned with differentiation and integration and the underlying concept is that of the limit.

- We studied the concepts of limits and rates of change:
  1. Limits – what a function, sequence or series tends to as a dependent variable approaches a given value
  2. Rates of change – slope of the function with respect to the independent variable

- Applications of limits and rates of change to heat transfer