Interactive Graphics Curves Using Parametric Equations

Dr. Niels Lobo

Computer Science

UCF EXCEL

Interactive Graphics Curves

Calculus Topic: Calculus with Parametric Curves

These are curves where the positions are specified in terms of a common parameter *t*, called Arclength.

Section 11.2 #3:

Find an equation of the tangent to the curve at the point

$$x = t^4 + 1, \quad y = t^3 + t, \quad t = -1$$

Calculus with Parametric Curves

$$x = t^4 + 1, \quad y = t^3 + t, \quad t = -1$$

Solution:

1) Compute Slope of Tangent, given by $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

2) Find point on curve as *(x,y)*, where tangent touches

3) Use point-slope form of line equation to get answer.

Summary of finding tangent

Three steps; and can swap order of steps 1 and 2, because they do not depend on each other.

Interactive Graphics Curves

Google “bezier curves``

- http://www.doc.ic.ac.uk/~dfg/AndysSplineTutorial/Beziers.html/

Interactive Graphics Curves

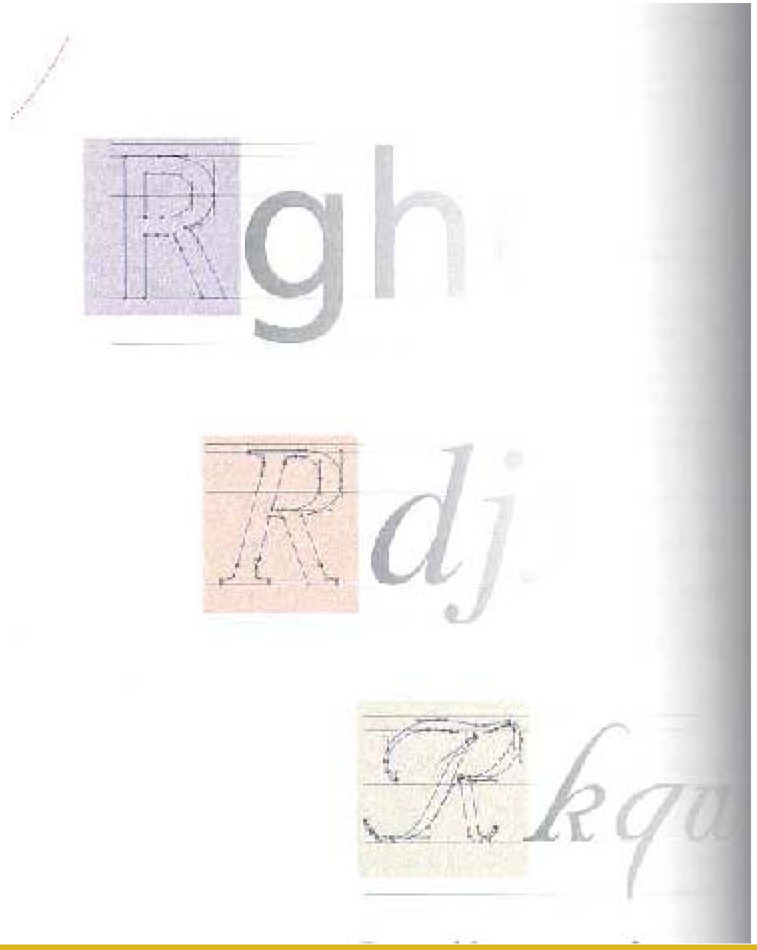
Uses:

-- Design of Fonts and other printer symbols

-- Consumer goods: shapes of cell phones, cars, etc.

Interactive Graphics Curves

Figure from Page 686.



Developer of Bezier Curves

Read page 705, Laboratory Project

Pierre Bezier (1910-1999) French mathematician

Bezier worked in automotive industry

Interactive Graphics Curves

Curve is specified by 2 equations:

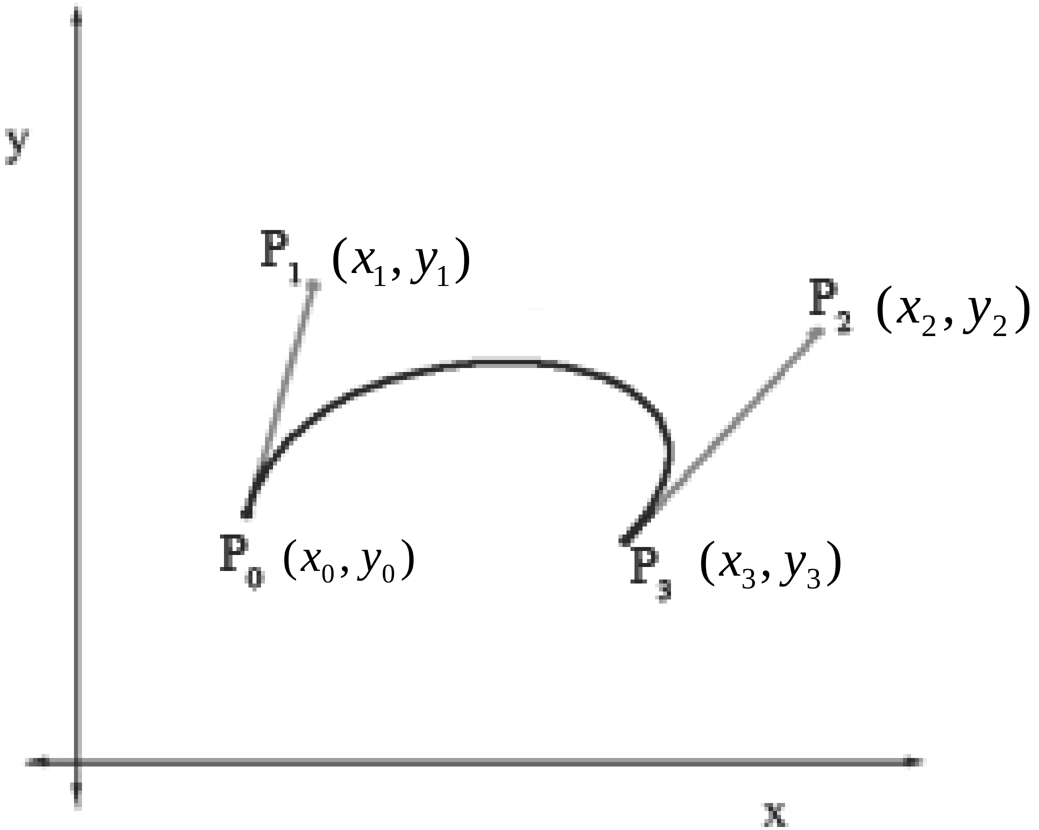
$$x = x_0(1-t)^3 + 3x_1 t(1-t)^2 + 3x_2 t^2(1-t) + x_3 t^3$$

$$y = y_0(1-t)^3 + 3y_1 t(1-t)^2 + 3y_2 t^2(1-t) + y_3 t^3$$

(x_0, y_0) and (x_3, y_3) are curve endpoints

(x_1, y_1) and (x_2, y_2) are guidepoints

A Bezier Curve



Interactive Graphics Curves

So, we specify the two endpoints, and the two guidepoints

Each endpoint-guidepoint pair is like a steering handle, to steer the curve.

Interactive Graphics Curves

- http://www.doc.ic.ac.uk/~dfg/AndysSplineTutorial/Beziers.html/

Interactive Graphics Curves

We will not examine how these equations came about or are derived;
that topic is covered in other classes, such as, a class on curves.

Interactive Graphics Curves

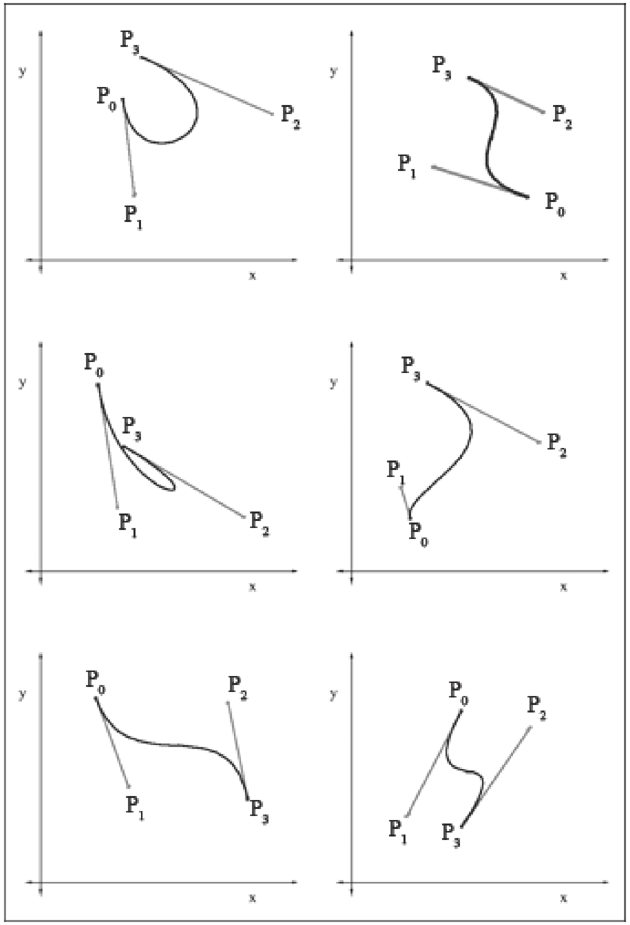
Properties of the Bezier Curve:

Slope of a handle is same as tangent at endpoint

Thus, rotating the handle forces the curve to follow the orientation of the handle.

Some example curves

Note that the handles are not part of the curve.



Interactive Graphics Curves

Mathematically, we will verify that:

Slope of a handle is same as tangent at endpoint

i.e., the tangent at (x_0, y_0) has same slope as the segment joining (x_0, y_0) to (x_1, y_1)

To show Handle Slope = Tangent

Tangent is given by $\dfrac{dx}{dy}$ which in turn is

given by: $\dfrac{dy/dt}{dx/dt}$

Our initial Bezier equation is

$$y = y_0(1-t)^3 + 3y_1t(1-t)^2 + 3y_2t^2(1-t) + y_3t^3$$

$$x = x_0(1-t)^3 + 3x_1t(1-t)^2 + 3x_2t^2(1-t) + x_3t^3$$

To show Handle Slope = Tangent

To find $\dfrac{dy/dt}{dx/dt}$, first get $\dfrac{dy}{dt}$

Given:

$$y = y_0(1-t)^3 + 3y_1 t(1-t)^2 + 3y_2 t^2(1-t) + y_3 t^3$$

$$\frac{dy}{dt} =$$

$$-3y_0(1-t)^2 + 3y_1(1-t)^2 - 6y_1 t(1-t) + 6y_2 t(1-t) - 3y_2 t^2 + 3y_3 t^2$$

To show Handle Slope = Tangent

Tangent is given by

$$\frac{dy/dt}{dx/dt} =$$

$$\frac{-3y_0(1-t)^2 + 3y_1(1-t)^2 - 6y_1t(1-t) + 6y_2t(1-t) - 3y_2t^2 + 3y_3t^2}{-3x_0(1-t)^2 + 3x_1(1-t)^2 - 6x_1t(1-t) + 6x_2t(1-t) - 3x_2t^2 + 3x_3t^2}$$

Interactive Graphics Curves

So, to get tangent at (x_0, y_0), we evaluate this expression

$$\frac{-3y_0(1-t)^2+3y_1(1-t)^2-6y_1t(1-t)+6y_2t(1-t)-3y_2t^2+3y_3t^2}{-3x_0(1-t)^2+3x_1(1-t)^2-6x_1t(1-t)+6x_2t(1-t)-3x_2t^2+3x_3t^2}$$

at $t=0$

To show Handle Slope = Tangent

So, to get tangent at (x_0, y_0), we evaluate this expression

$$\frac{-3y_0(1-t)^2+3y_1(1-t)^2-6y_1t(1-t)+6y_2t(1-t)-3y_2t^2+3y_3t^2}{-3x_0(1-t)^2+3x_1(1-t)^2-6x_1t(1-t)+6x_2t(1-t)-3x_2t^2+3x_3t^2}$$

at $t=0$

And got the answer, …

To show Handle Slope = Tangent

So, at $t = 0$ our slope simplifies to $\dfrac{y_1 - y_0}{x_1 - x_0}$,

But, this is the definition of the slope of a segment joining two points (x_0, y_0) and (x_1, y_1)

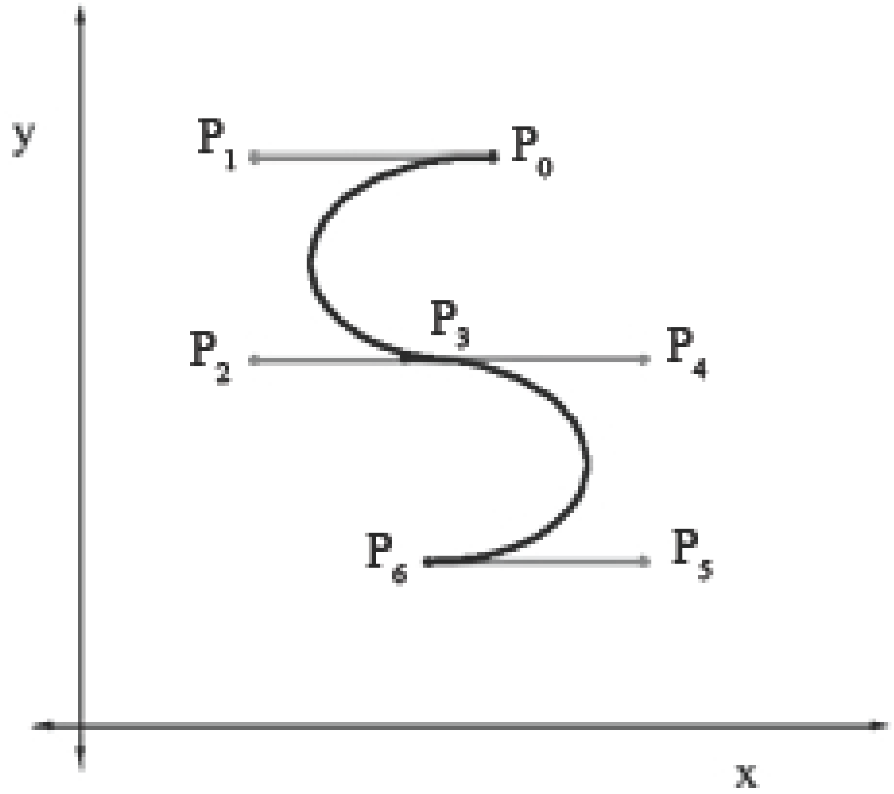
So, we have shown that slope of handle (segment) is same as slope of tangent at the endpoint (x_0, y_0)

Joining Bezier Curves

We can join two (or more) curves end-to-end to compose complicated curves.

Joining Bezier Curves

Can join curves smoothly by forcing handles at join location to agree in their slopes, but point in opposite directions.



Revisit the Bezier Curve