

*Applying hyperbolic functions to
quantum tunneling and
electromagnetic wave problems in
physics and engineering (Lecture 2)*

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Feb. 4, 2008



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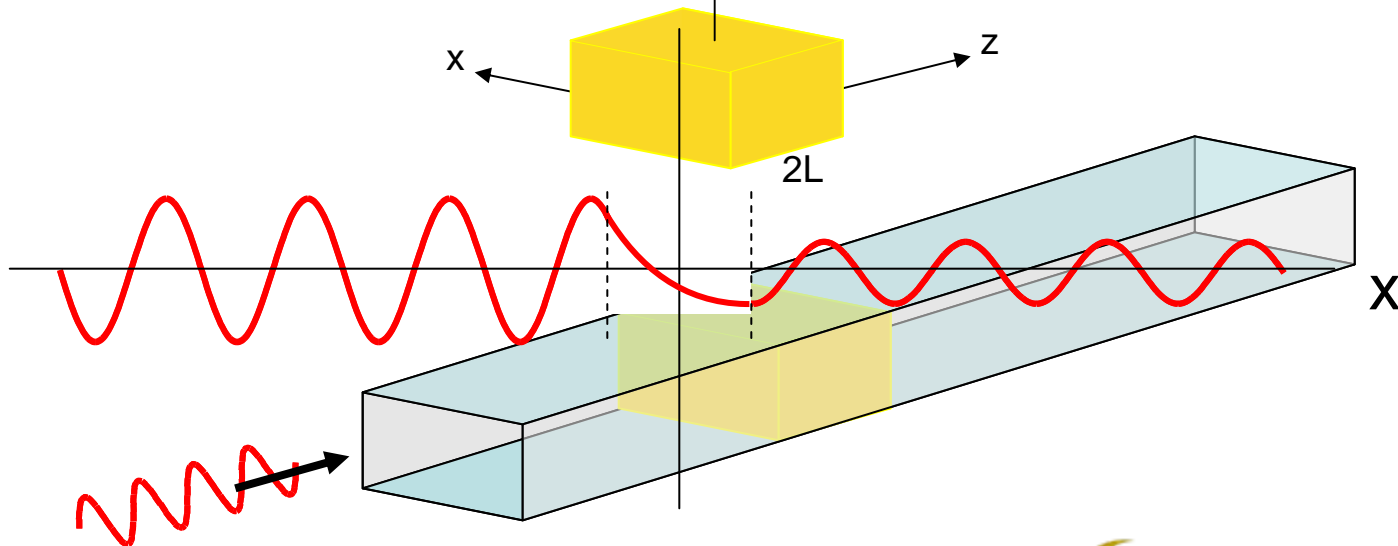
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What our goal is this afternoon

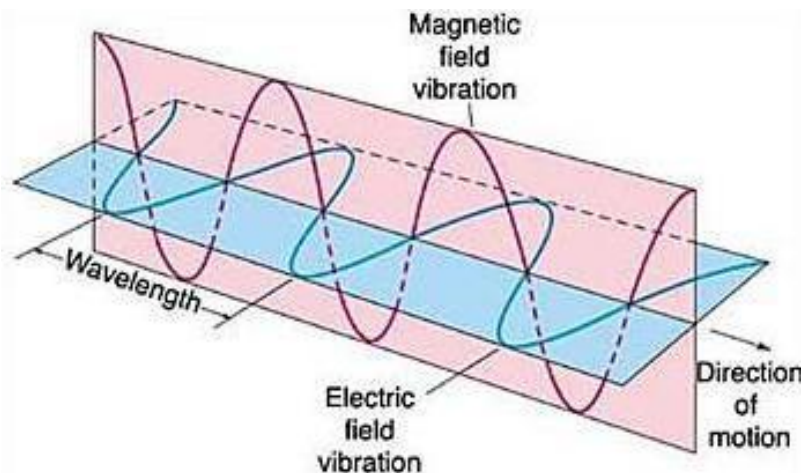
Penetrate material with electromagnetic waves that reshape into hyperbolic functions. (*finish*)
and

Violate conservation of energy in order to quantum tunnel with a quantum wave shaped like hyperbolic functions.



$$\mathbf{E}'' = \frac{1}{c^2} \frac{d^2}{dt^2} \mathbf{E}$$

- A. The relationship for radiation involves the electric field (E) and magnetic field (B)
- B. They are coupled
- C. In space-time
- D. With second derivative in space and in time.
 - 1) $\sin(kx \pm \omega t)$
 - 2) $\cos(kx \pm \omega t)$
 - 3) Dispersion relation



$$c^2 = \frac{\omega^2}{k^2}$$

- E. We extract this physical meaning with mathematical analysis!
- ✓ SWEEEEEEEEET



$$\mathbf{E}'' = \frac{1}{c^2} \frac{d^2}{dt^2} \mathbf{E}$$

6. Sines and cosines of time and space

$$\mathbf{E}'' = -\mathbf{k}^2 \mathbf{E}$$

7. Koshers ω and k , if...

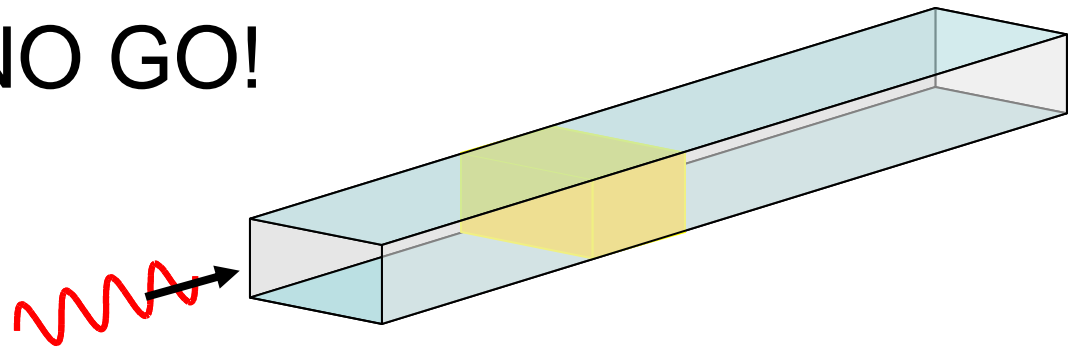
$$\frac{d^2}{dt^2} \mathbf{E} = -\omega^2 \mathbf{E}$$



A. Inside material, the physics changes.

- 1) Light moves more slowly.
- 2) Energy is absorbed from E and B.
- 3) Heat flows, outer surfaces cool off
- 4) New spatial and temporal derivatives for E and B fields.

B. sine, cosine = NO GO!



$$E'' = \cancel{\frac{1}{c^2}} \frac{d^2}{dt^2} E$$

C. The old derivatives relationship changes, i.e., for $E_z \dots$

$$E''_z - h^2 E_z = 0$$

$$E''_z = +h^2 E_z$$

D. New deriv. rel. means new functions do the work.

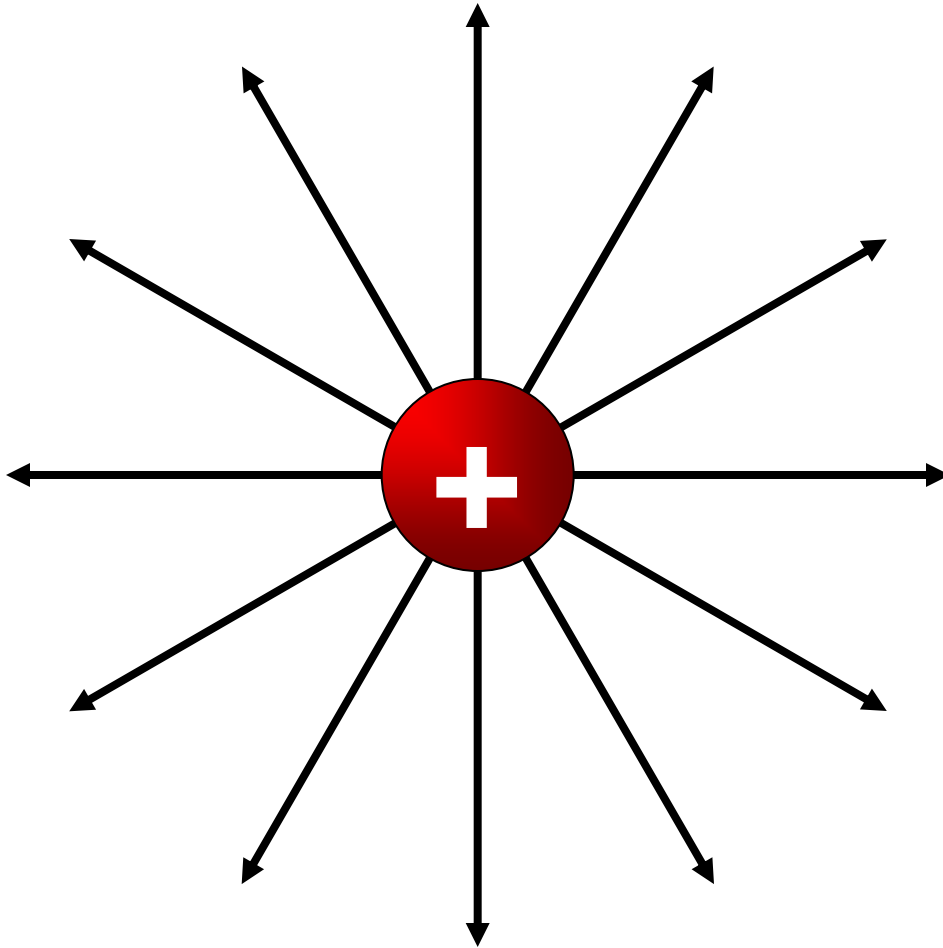
E. Hyperbolic functions, $\cosh(u)$ and $\sinh(u)$.





- Using I-Clicker
- Building on lecture last Monday.
- Pointing toward our objectives for today: penetrating radiation and quantum tunneling





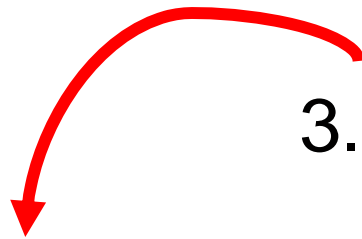
$$\mathbf{E}_z'' - h^2 \mathbf{E}_z = 0$$

$$\mathbf{E}_z'' = +h^2 \mathbf{E}_z$$

$$\mathbf{u}_z'' - h^2 \mathbf{u}_z = 0$$

$$\mathbf{u}_z'' = +h^2 \mathbf{u}_z$$

1. Electric field component in direction of propagation, z.
2. Maxwell equation modifies to this one.
3. So use $\cosh(az)$ and $\sinh(az)$



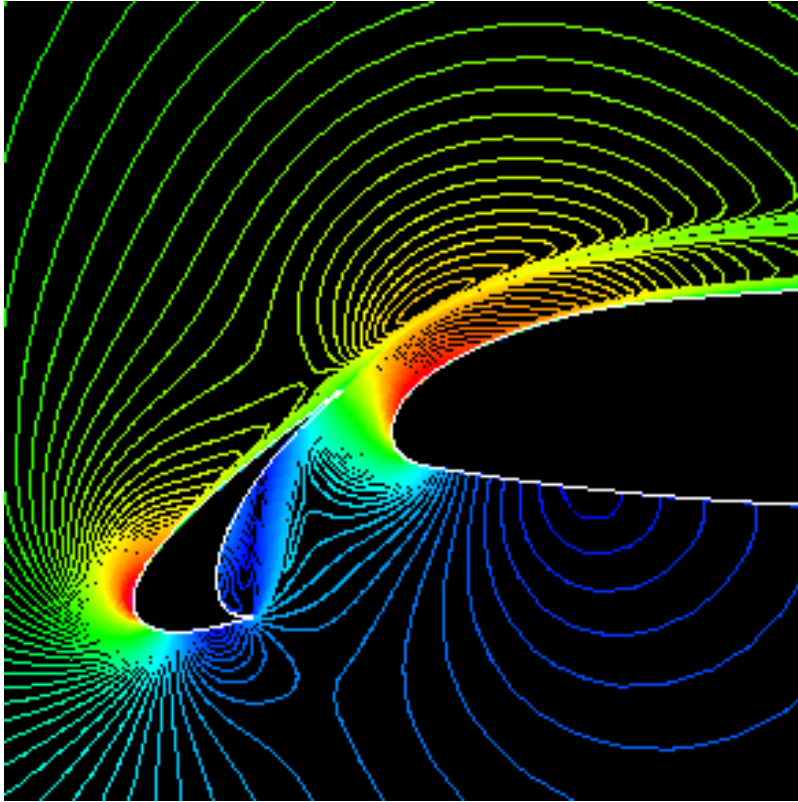
$$u(x, y, z) = \cos\left(\frac{\pi x}{2nL}\right) \cos\left(\frac{\pi y}{2mL}\right) [A \cosh(az) + B \sinh(az)]$$





4. That function $u(x,y,z)$ goes into the Maxwell equation
5. Result: conditions that must hold
 - a) Spatial factor a
 - b) Dispersion relation
 - c) Generic combination factors A, B
 - d) Boundary matching





5. Result: conditions that must hold
 - a) Spatial factor a
 - b) Dispersion relation
 - c) Generic combination factors A, B
 - d) Boundary matching
6. This is where the work comes in, and lots of discoveries.

E.g., boundary conditions must allow for attenuation: E_z may $\rightarrow 0$ at back end.



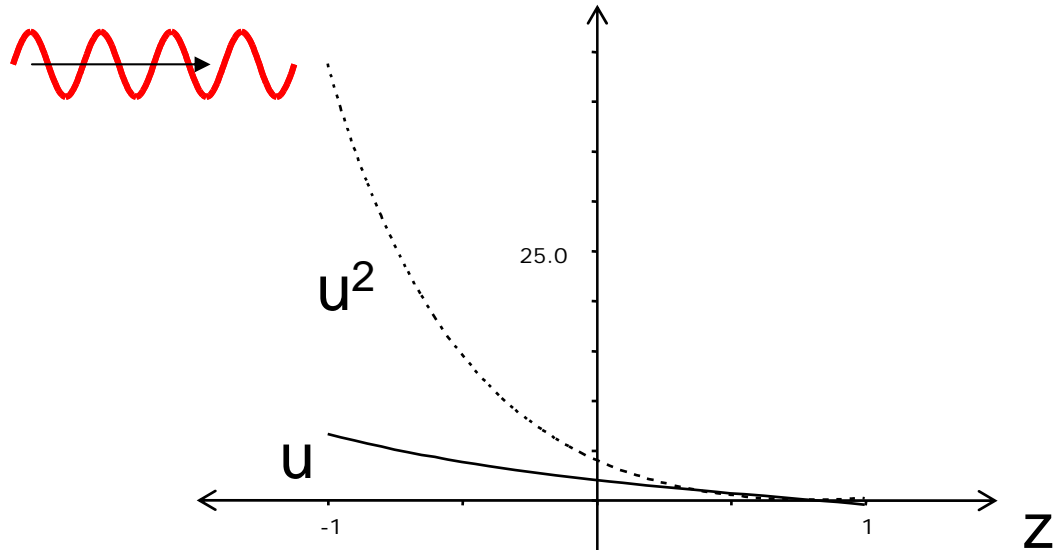
11. Note: $\cosh(az)$ and $\sinh(az)$ must be combined for this case.

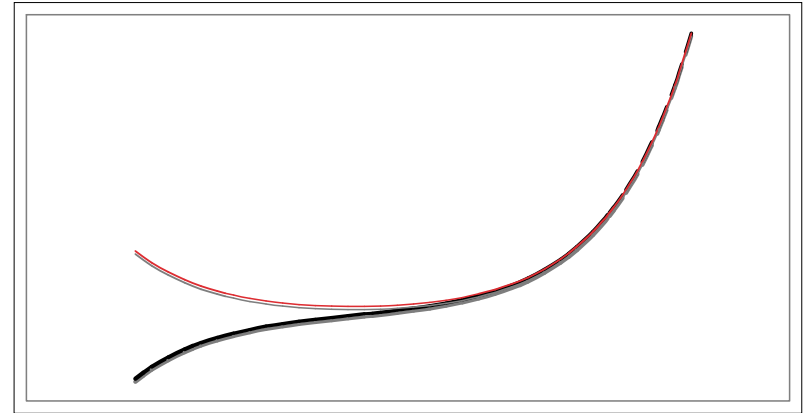
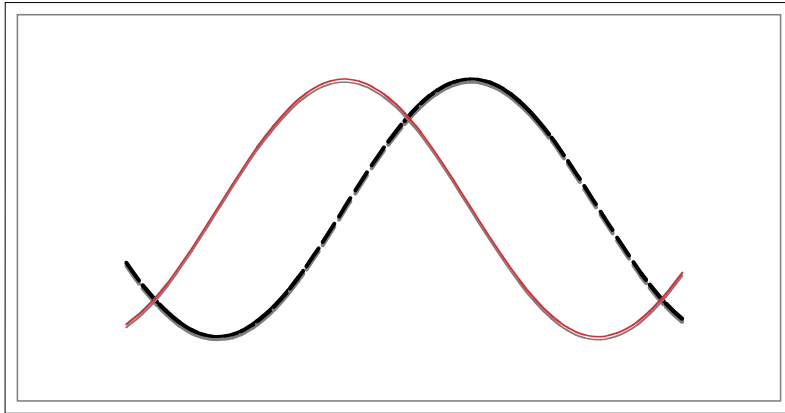
12. $A \cosh(az) + B \sinh(az)$

✓ $A = 2$

✓ $B = -3$

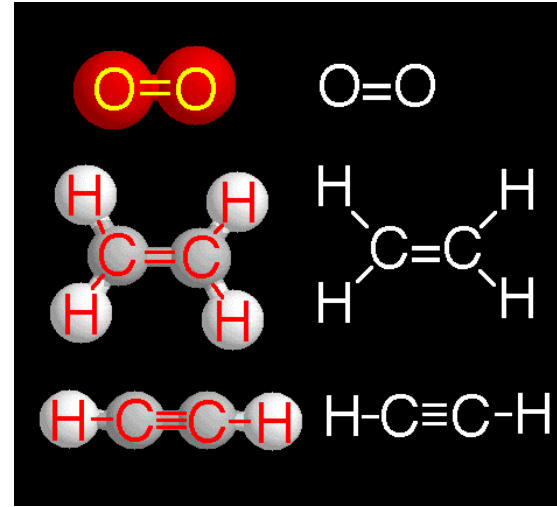
✓ $a = 1$





- A. The shape of a function and its derivative are filled with physical meaning.
- B. You must develop the eyes to spot the meaning, or
- C. To place meaning into the model you use.





- **Violate conservation of energy in order to quantum tunnel with a quantum wave shaped like hyperbolic functions.**

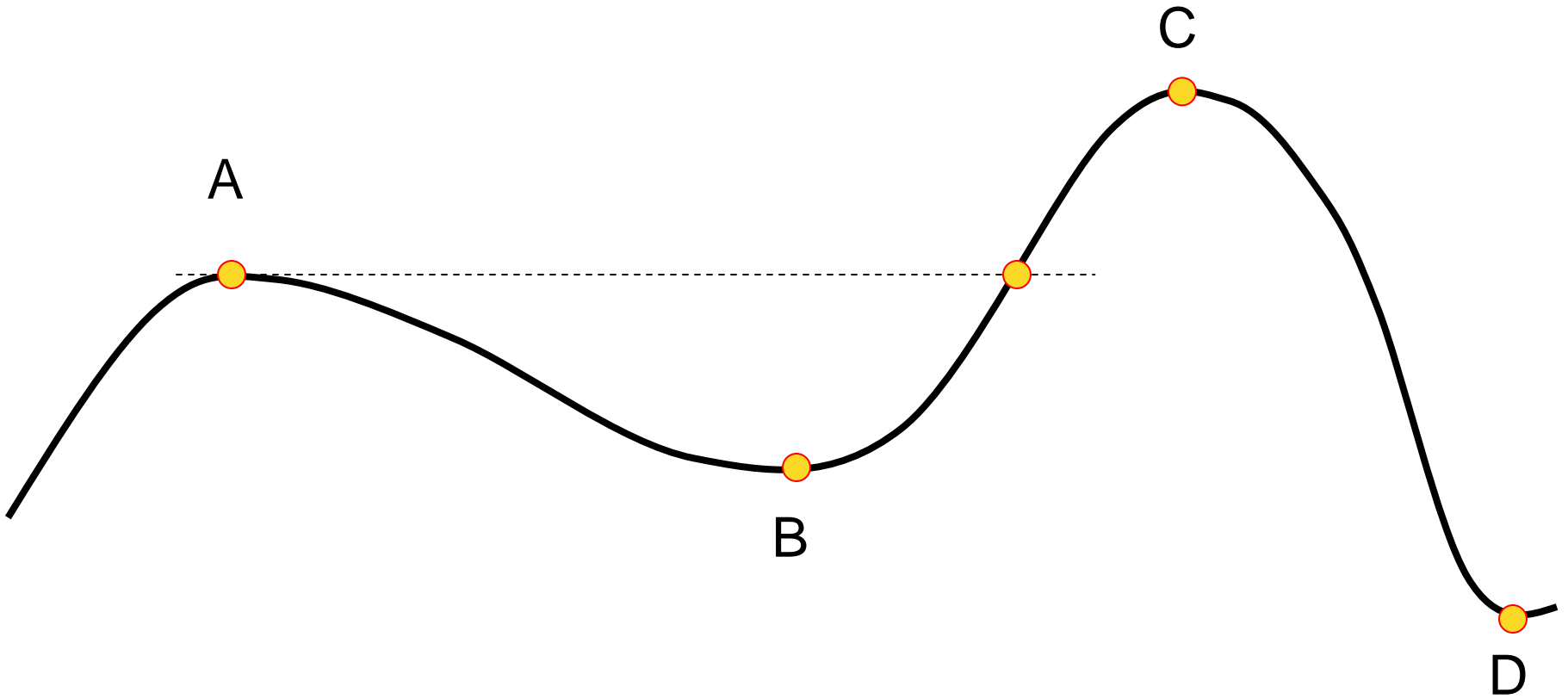


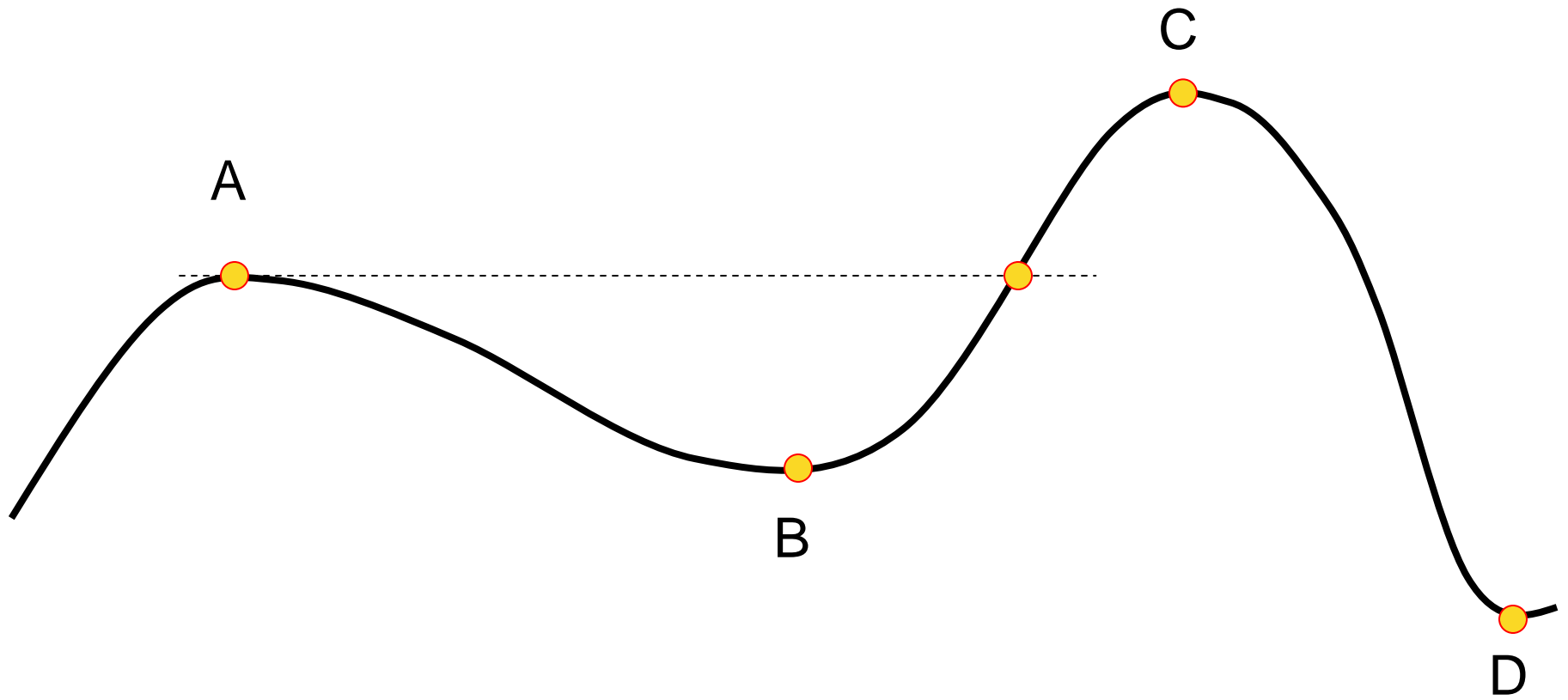
NOW a new question...



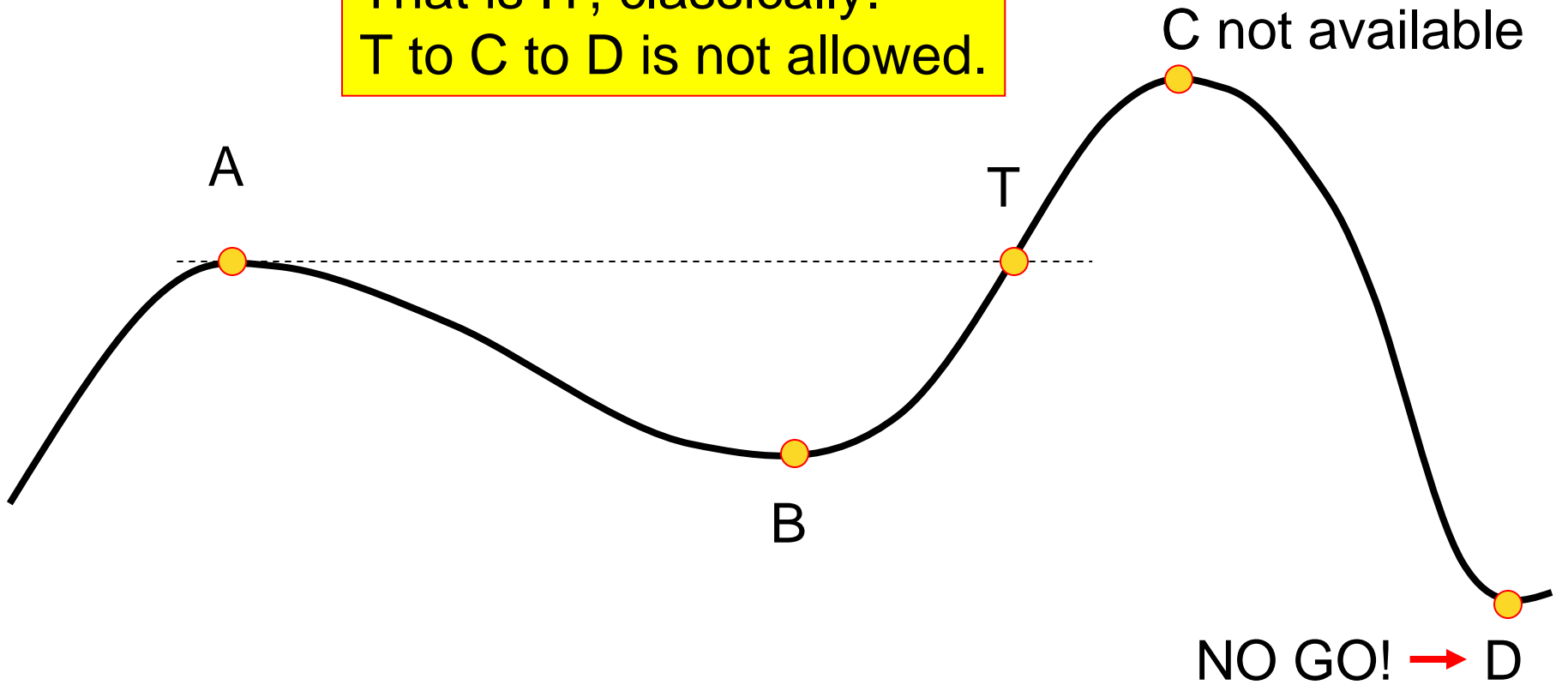
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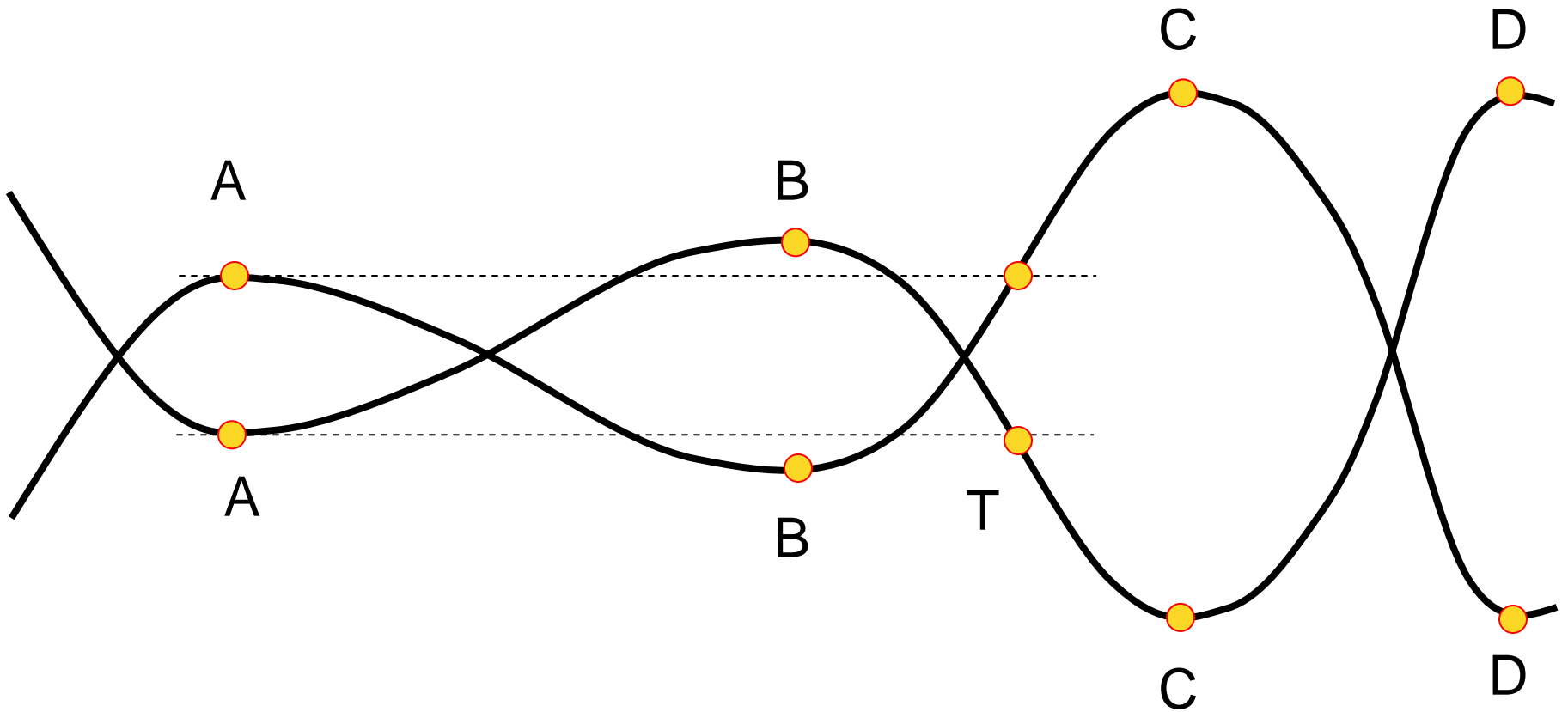


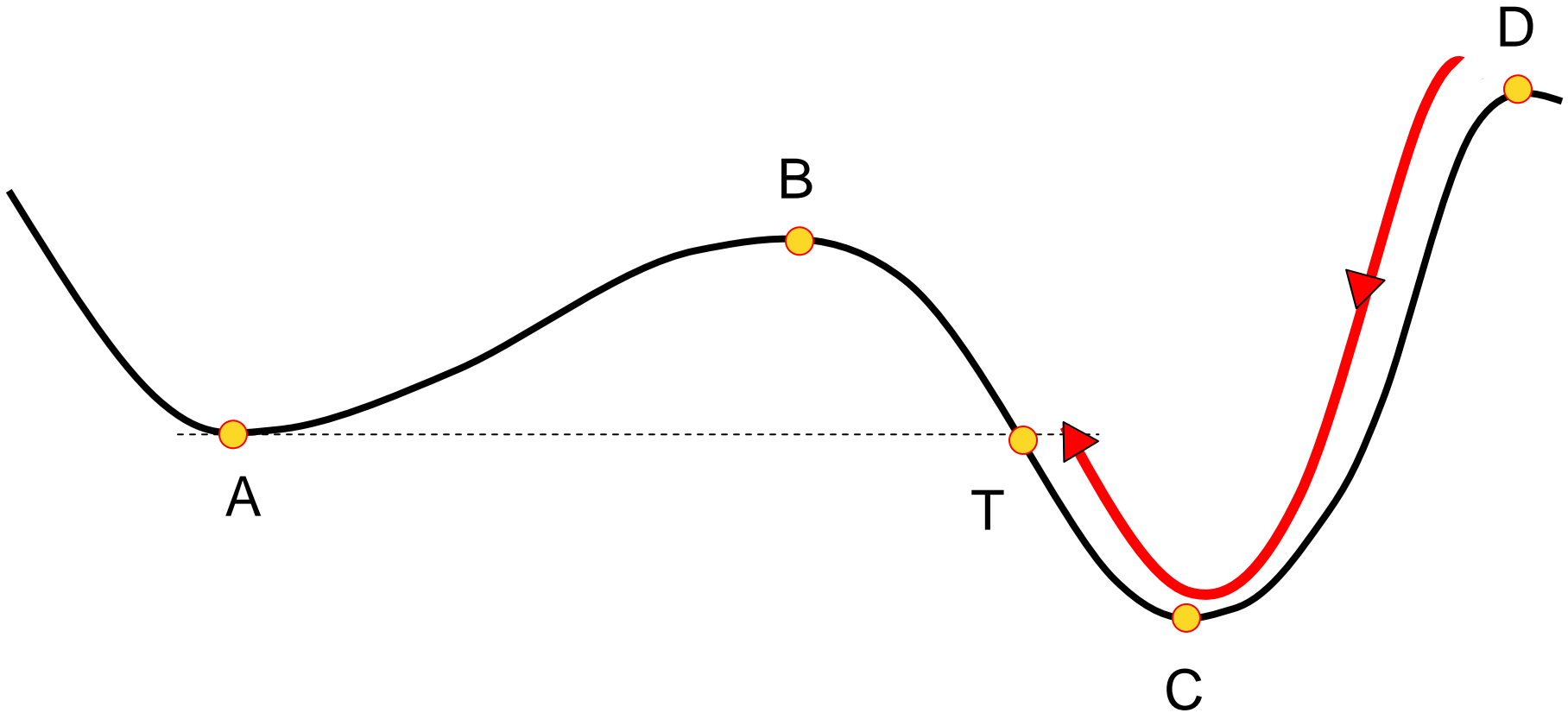


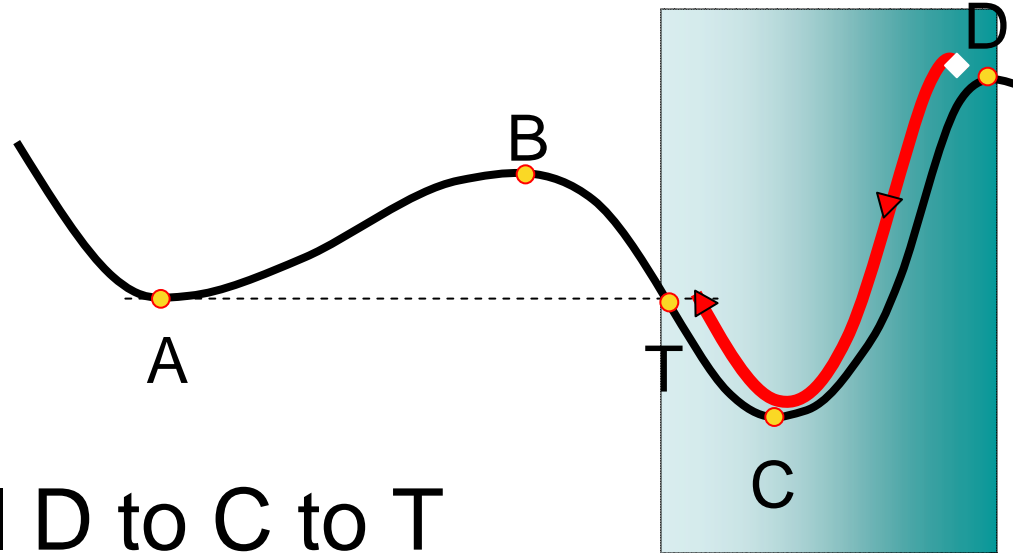


That is IT, classically.
T to C to D is not allowed.





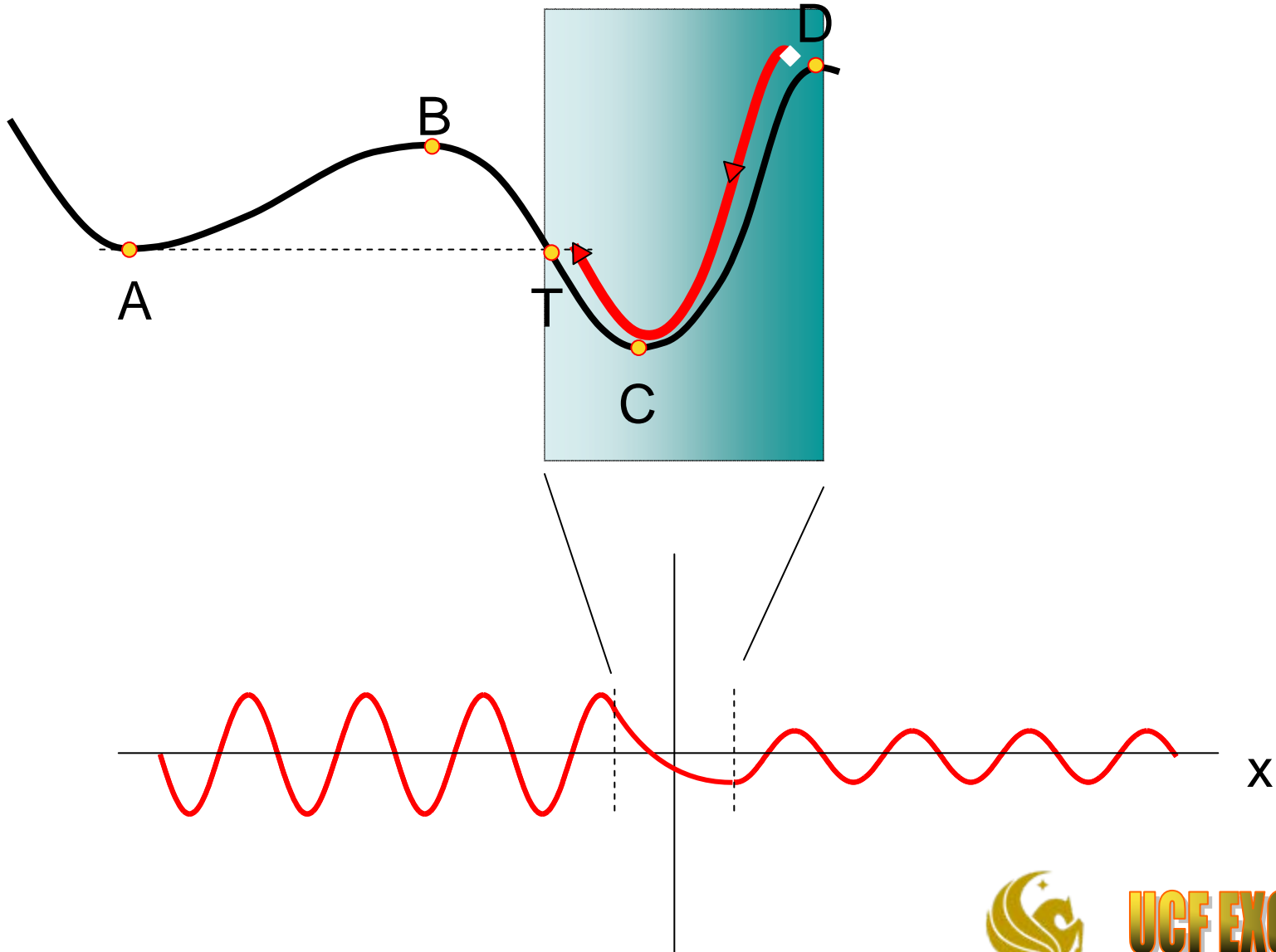




1. We model D to C to T
2. Rotate time axis t to i .
3. A bit like $F = ima$ or negative energy!
4. Hyperbolic cosine and hyperbolic sine are THE tools!



Inside the barrier, cosh and sinh!





When you are using good old cosh and sinh, remember that you've got the tool that rotates time in order to model quantum tunneling.



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