Applying hyperbolic functions to quantum tunneling and electromagnetic wave problems in physics and engineering (Lecture 1)

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What our goal is this afternoon

Penetrate material with electromagnetic waves that reshape into hyperbolic functions.

Ch. 2 in Applications of Calculus II
What are we talking about?
What are we talking about?

Weird…but handy in physics!

cosh

sinh
Recall: Euler’s formula...

We need this today!

\[ e^{iz} = \cos(z) + i \sin(z) \]

\[ \cos(z) = \frac{1}{2} \left( e^{iz} + e^{-iz} \right) \]

\[ \sin(z) = \frac{1}{2} \left( e^{iz} - e^{-iz} \right) \]
Modify for today’s topic

Mutatis mutandam…

\[
\cosh(z) = \frac{1}{2} \left( e^{iz} + e^{-iz} \right)
\]

\[
\sinh(z) = \frac{1}{2} \left( e^{iz} - e^{-iz} \right)
\]
Let’s do some warmups

- Using I-Clicker
- Building on Prof. Self’s talk two Monday’s ago
- Pointing toward our objective for today: penetrating radiation.
What we are talking about

Each graphed over \([-1, 2]\) for each

\[ \cos(kx), \sin(kx) \]

\[ \cosh(kx), \sinh(kx) \]

\[ \exp(kx), \exp(-kx) \]

\( k = 2.7 \) for each
Here we go. Let’s look at derivatives.

\[
\frac{d}{dt} \cos(\omega t) = -\omega \sin(\omega t) \quad \frac{d}{dt} \sin(\omega t) = \omega \cos(\omega t)
\]

In general: after two diffs you get back to same function, 
...but with an extra factor of \(-\omega^2\).

\[
\frac{d^2}{dt^2} \cos(\omega t) = -\omega^2 \cos(\omega t) \quad \frac{d^2}{dt^2} \sin(\omega t) = -\omega^2 \sin(\omega t)
\]
General derivatives relationships here:

\[
\frac{d^2}{dt^2} \left[ f(\omega t) \right] = -\omega^2 f(\omega t)
\]

In general: after two diff’s you get back to same function, …but with an extra factor of \(-\omega^2\). No sweat with \(-\omega^2\).

\[
\frac{d^2}{dt^2} \left[ f(\omega t) \right] + \omega^2 f(\omega t) = 0
\]
No sweat because...

1. That kind of derivative relationship is needed in spring systems, e.g.,
   ✓ Oscillators
   ✓ Vibrations

2. $F = -kx$ in springs

3. $F = ma$...as always

4. $a = \text{second deriv. of } x$!
Brainiac iClicker question coming…

\[ m \ a = -k \ x \]

\[ m \ \frac{d^2}{dt^2} (x[t]) = -k \ x(t) \]
Onward to radiation

1. The relationship for radiation involves the electric field (E) and magnetic field (B) in vacuum.

\[ E'' = \frac{1}{c^2} \frac{d^2}{dt^2} E \]

2. They are coupled.
3. In space-time.
4. With second derivative in space and in time.
5. Wiggles that move.
Spacetime implications  

6. Sines and cosines of time and space
7. Sin(kx ± \omega t)
8. Cos(kx ± \omega t)
9. Kosher \omega and k, if…

\[
\frac{d^2}{dt^2} E = -\omega^2 E
\]

\[
E'' = \frac{1}{c^2} \frac{d^2}{dt^2} E
\]

\[
E'' = -k^2 E
\]
Another brainiac question coming…

\[ E'' = -k^2 E \]

\[ \frac{d^2}{dt^2} E = -\omega^2 E \]
What is this physical relationship to $c$?

A. Dispersion relation (fancy terminology)

B. Refraction of light (transmission)
   ✓ E.g., prisms disperse the colors of sunlight

C. Absorption of light
   ✓ E.g., Superman, lead absorbs x-rays.

$$c^2 = \frac{\omega^2}{k^2}$$
Penetrating a material with radiation

1. Inside material, the physics changes.
2. Light moves more slowly.
3. Energy is absorbed from $E$ and $B$.
4. Heat flows, outer surfaces cool off.
5. New spatial and temporal derivatives for $E$ and $B$ fields.

» sine, cosine = NO GO!
Now a new derivatives relation must hold.

\[ E'' = \frac{1}{c^2} \frac{d^2}{dt^2} E \]

6. The old derivatives relationship changes, i.e., for \( E_z \)

\[ E''_z - h^2 E_z = 0 \]

\[ E''_z = +h^2 E_z \]

7. New deriv. rel. means new functions do the work.

8. Hyperbolic functions, \( \cosh(u) \) and \( \sinh(u) \).