Applying hyperbolic functions to quantum tunneling and electromagnetic wave problems in physics and engineering (Lecture 1)

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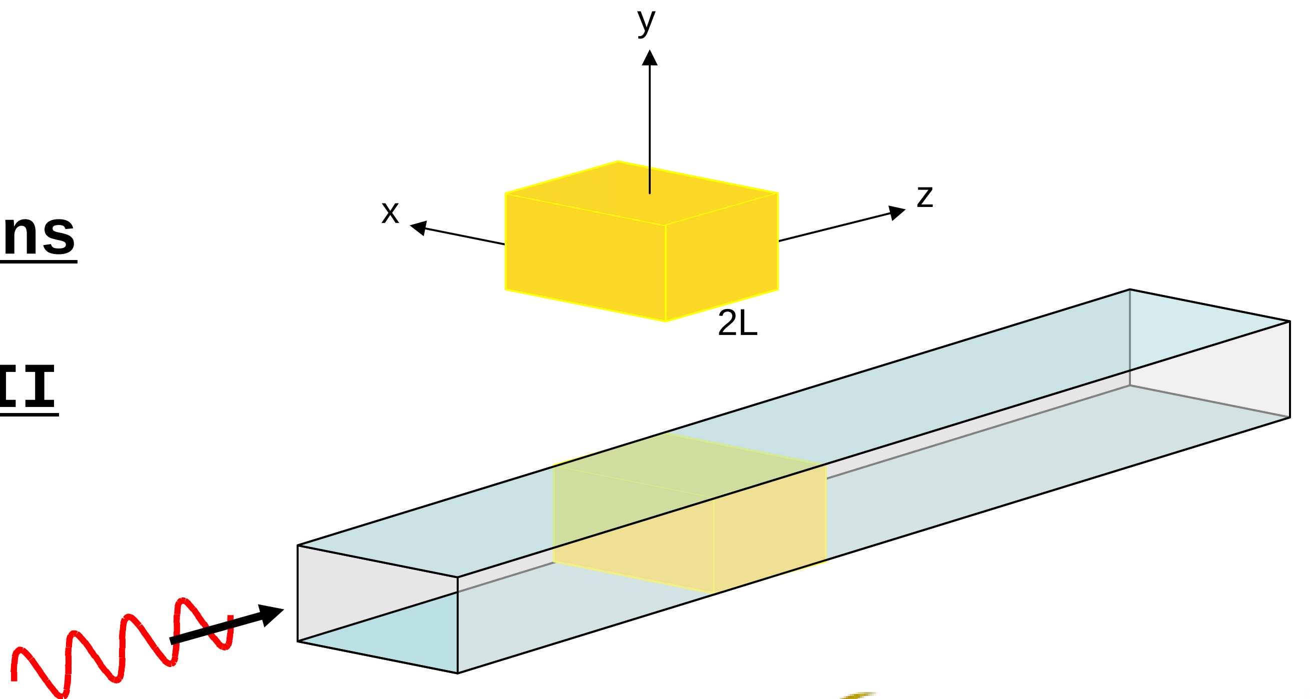
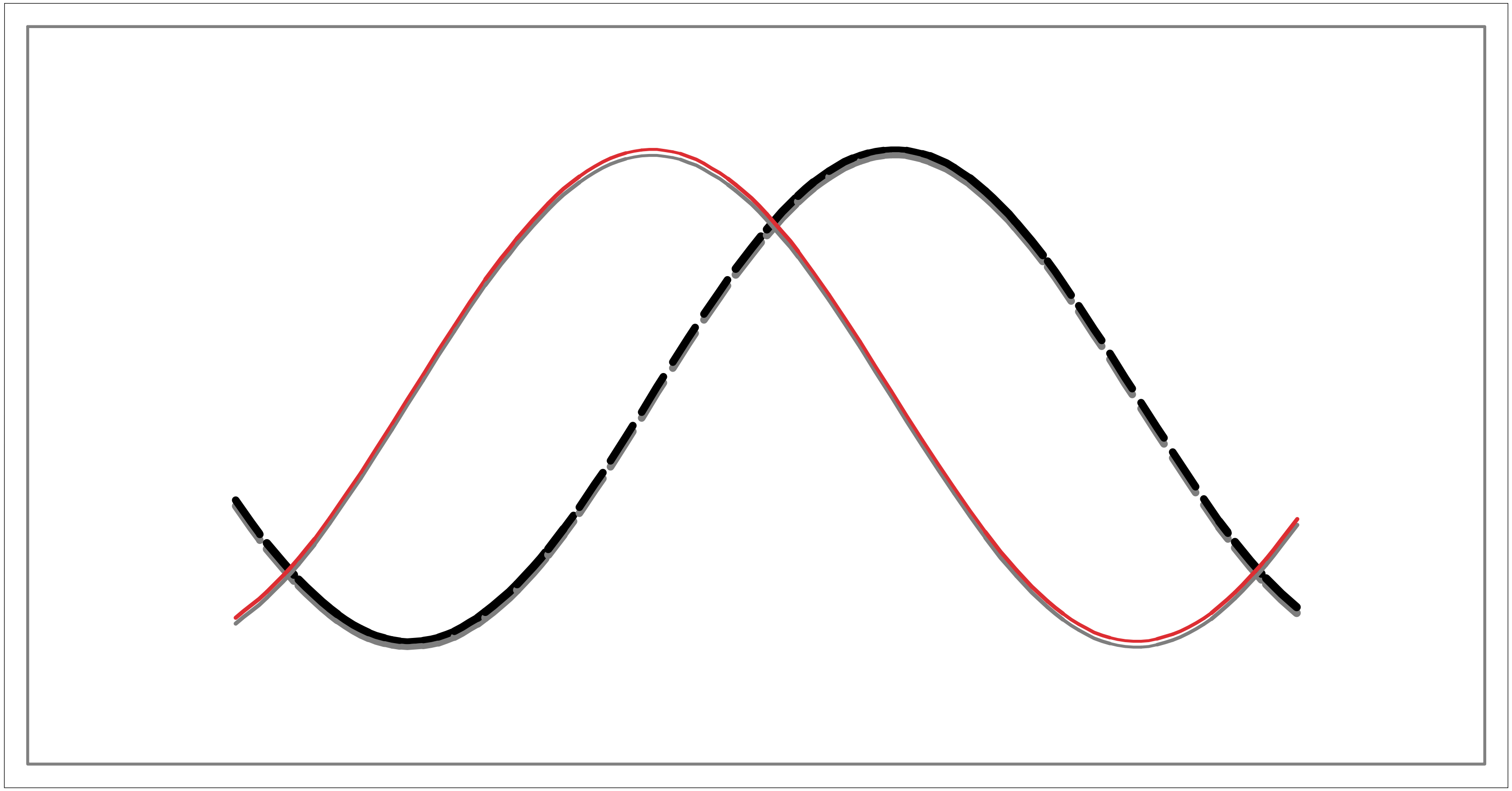
What our goal is this afternoon

Penetrate material with electromagnetic waves that reshape into hyperbolic functions.

Ch. 2 in
<u>Applications of Calculus II</u>



What are we talking about?



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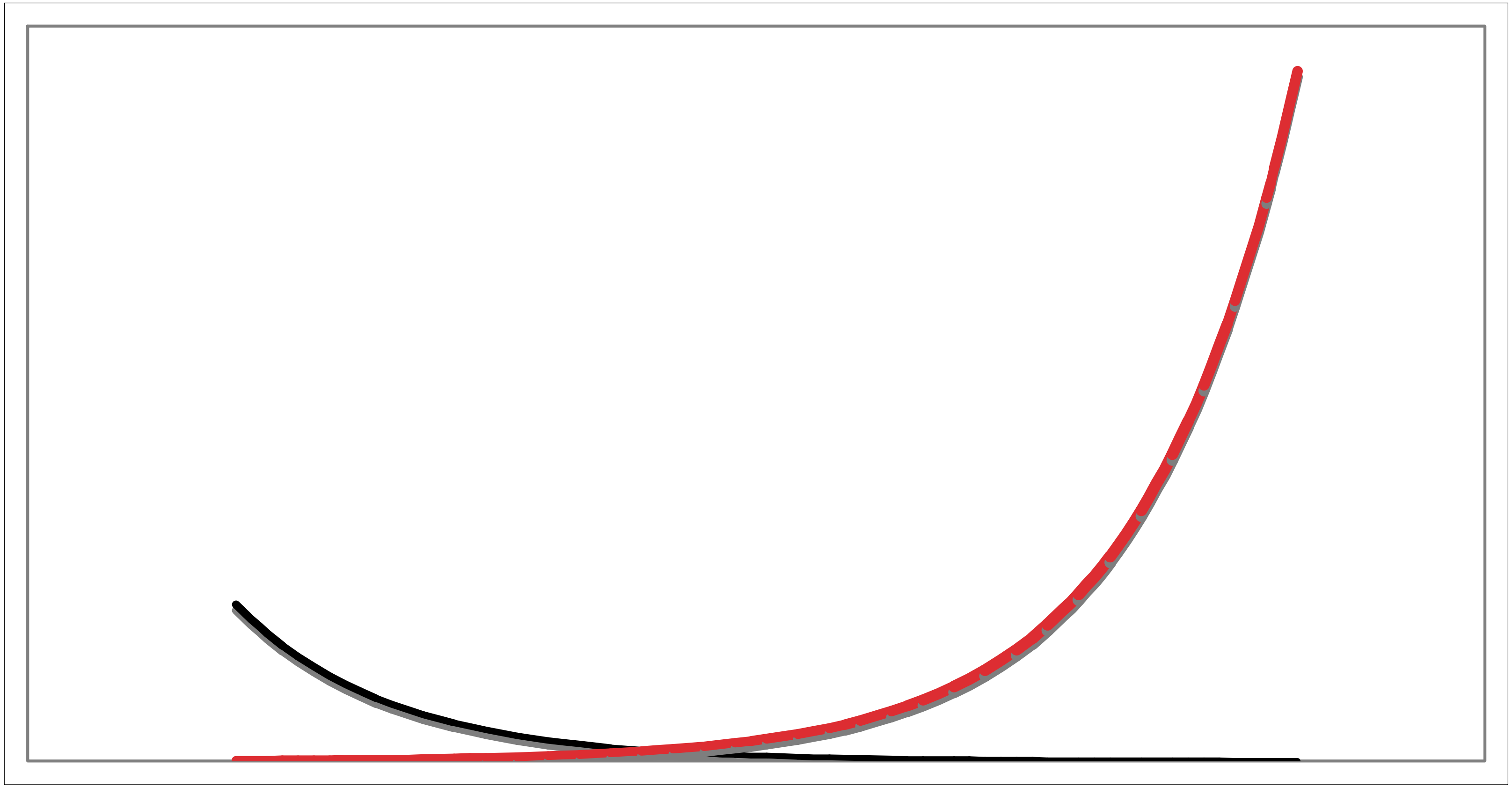

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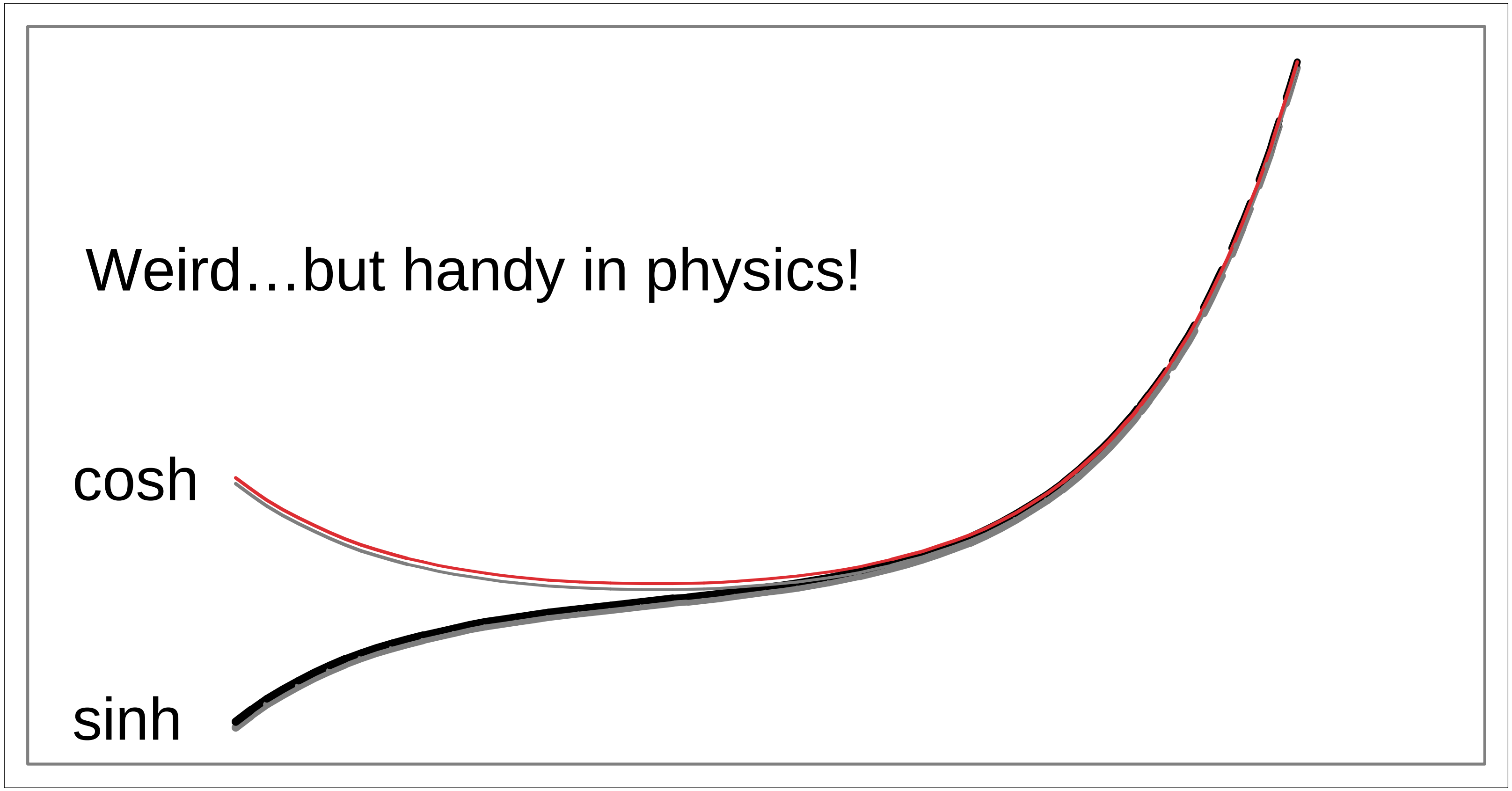

Recall: Euler's formula…

We need this today!

$$e^{iz} = \cos(z) + i\sin(z)$$

$$\cos(z) = \tfrac{1}{2}\left(e^{iz} + e^{-iz}\right)$$

$$\sin(z) = \tfrac{1}{2}\left(e^{iz} - e^{-iz}\right)$$

Mutatis mutandam…

$$\cosh(z) = \tfrac{1}{2}\left(e^{z} + e^{-z}\right)$$

$$\sinh(z) = \tfrac{1}{2}\left(e^{z} - e^{-z}\right)$$

Let’s do some warmups

- Using I-Clicker
- Building on Prof. Self’s talk two Monday’s ago
- Pointing toward our objective for today: penetrating radiation.

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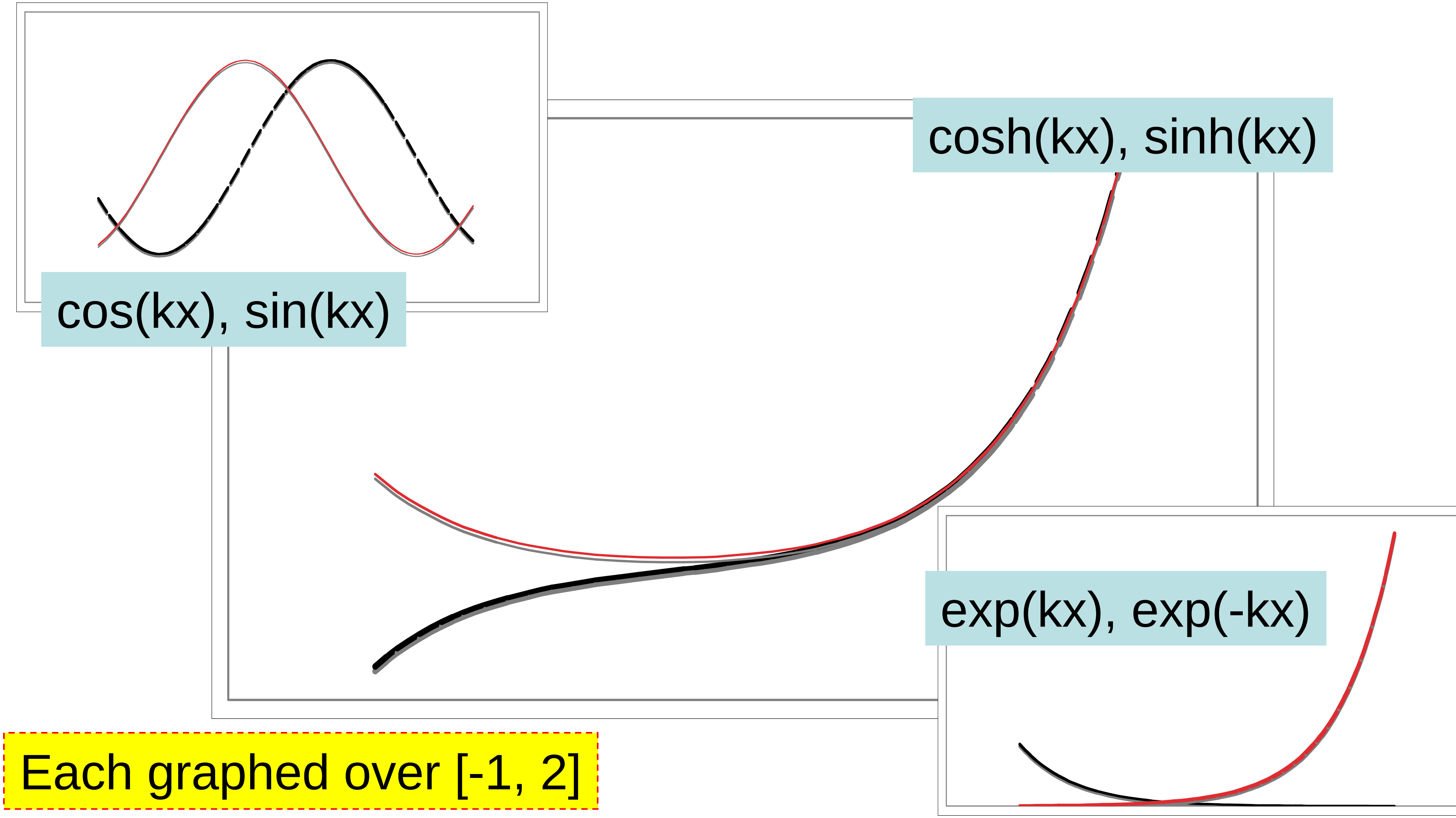
What we are talking about



Each graphed over [-1, 2]

k = 2.7 for each

Here we go. Let's look at derivatives.

$$\frac{d}{dt}\left[\cos(\omega t)\right] = -\omega \sin(\omega t)$$

$$\frac{d}{dt}\left[\sin(\omega t)\right] = \omega \cos(\omega t)$$

In general: after two diff's you get back to same function,
…but with an extra factor of $-\omega^2$.

$$\frac{d^2}{dt^2}\left[\cos(\omega t)\right] = -\omega^2 \cos(\omega t)$$

$$\frac{d^2}{dt^2}\left[\sin(\omega t)\right] = -\omega^2 \sin(\omega t)$$

General derivatives relationships here:

$$\frac{d^2}{dt^2}\left[f(\omega t)\right] = -\omega^2 f(\omega t)$$

In general: after two diff's you get back to same function,
…but with an extra factor of $-\omega^2$. No sweat with $-\omega^2$..

$$\frac{d^2}{dt^2}\left[f(\omega t)\right] + \omega^2 f(\omega t) = 0$$

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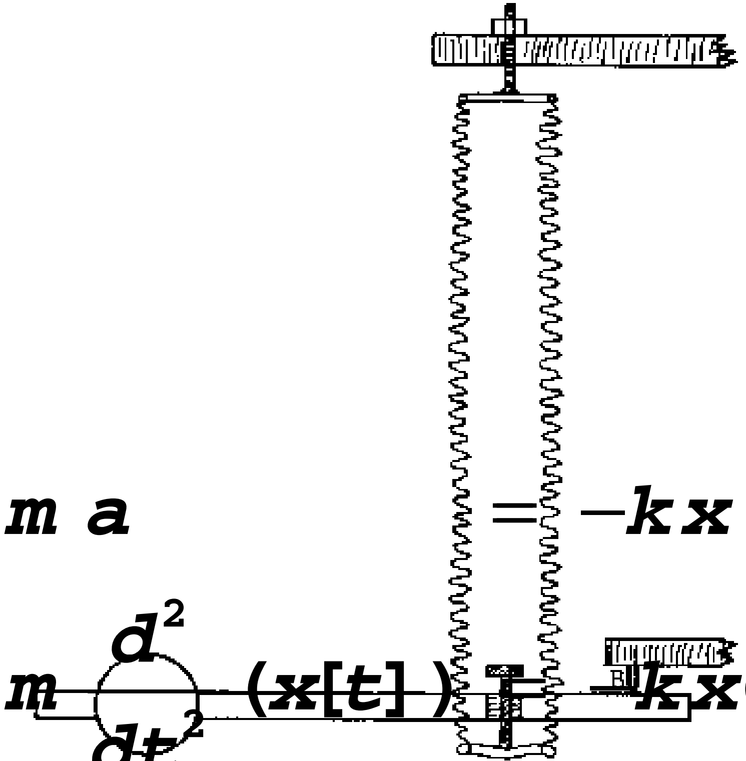
NSF

No sweat because…

1. That kind of derivative relationship is needed in spring systems, e.g.,
   - ✓ Oscillators
   - ✓ Vibrations
2. F = -kx in springs
3. F = ma…as always
4. a = second deriv. of x!



$$m\,a = -k\,x$$

$$m \frac{d^2}{dt^2}(x[t]) = -k\,x(t)$$

Simple seismometer

Brainiac iClicker question coming…

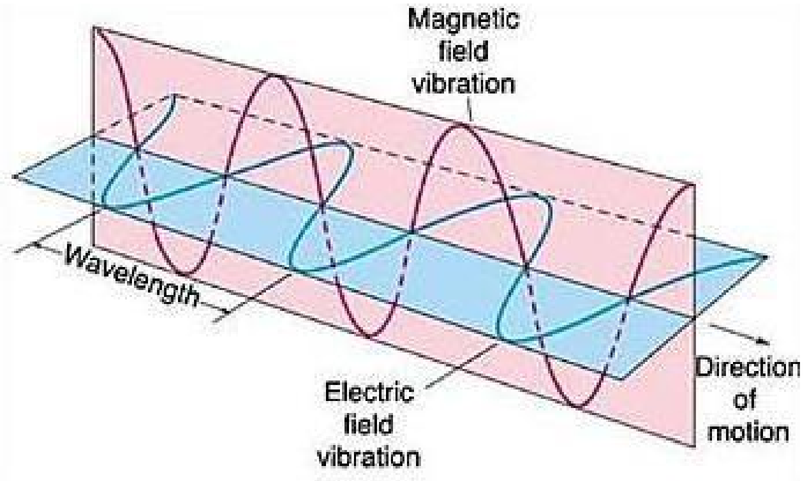
$$m\,a = -k\,x$$

$$m\,\frac{d^2}{dt^2}\left(x[t]\right) = -k\,x(t)$$

Onward to radiation

$$E'' = \frac{1}{c^2}\frac{d^2}{dt^2}E$$

in vacuum



1. The relationship for radiation involves the electric field (E) and magnetic field (B)
2. They are coupled
3. In space-time
4. With second derivative in space and in time.
5. Wiggles that move

Spacetime implications

$$E'' = \frac{1}{c^2}\frac{d^2}{dt^2}E$$

$$E'' = -k^2 E$$

$$\frac{d^2}{dt^2}E = -\omega^2 E$$

6. Sines and cosines of time and space
7. Sin(kx ± ωt)
8. Cos(kx ± ωt)
9. Kosher ω and k, if…

Another brainiac question coming…

$$E'' = -k^2 E$$

$$\frac{d^2}{dt^2} E = -\omega^2 E$$

What is this physical relationship to c?

A. Dispersion relation (fancy terminology)

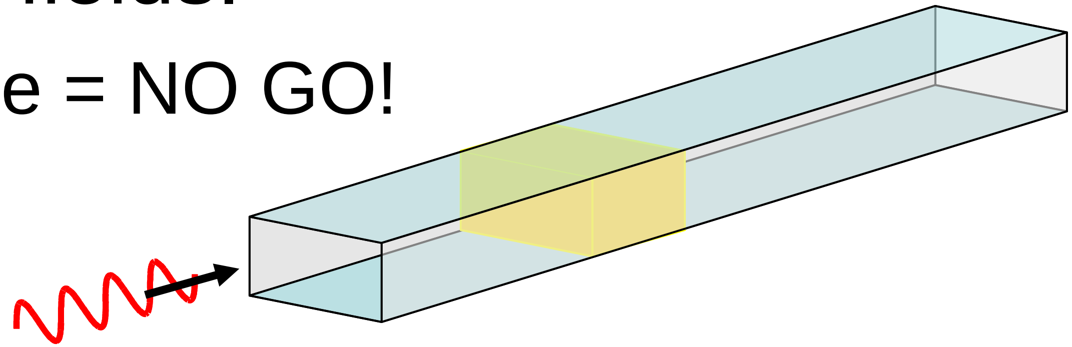
B. Refraction of light (transmission)
   - ✓ E.g., prisms disperse the colors of sunlight

C. Absorption of light
   - ✓ E.g., Superman, lead absorbs xrays.

$$c^2 = \frac{\omega^2}{k^2}$$

Penetrating a material with radiation

1. Inside material, the physics changes.
2. Light moves more slowly.
3. Energy is absorbed from E and B.
4. Heat flows, outer surfaces cool off
5. New spatial and temporal derivatives for E and B fields.
   » sine, cosine = NO GO!



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Now a new derivatives relation must hold.

$$E'' = \frac{1}{c^2}\frac{d^2}{dt^2}E$$

6. The old derivatives relationship changes, i.e.., for E_z...

$$E_z'' - h^2 E_z = 0$$

$$E_z'' = +h^2 E_z$$

7. New deriv. rel. means new functions do the work.
8. Hyperbolic functions, cosh(u) and sinh(u).