Rates of Change

by
Andrew Winningham
UCF EXCEL Applications of Calculus

Connections to Other Courses

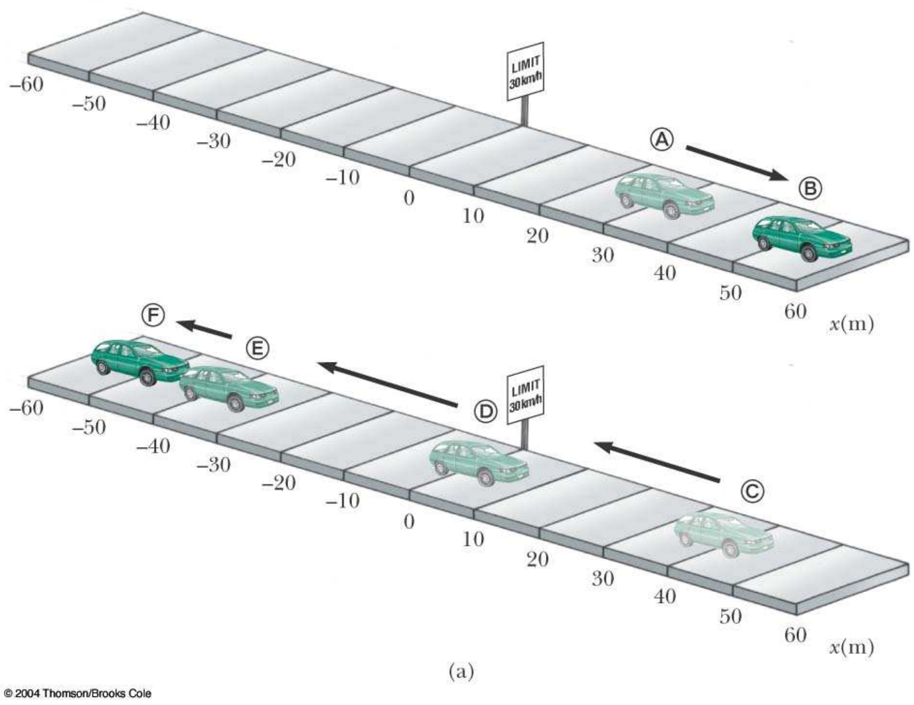
- Anything interesting involves change
- If something changes, we can also discuss how fast it is changing, its rate of change
- These concepts occur in all the technical fields
- We're going to talk about kinematics (PHY 2048) as a real world example of rates of change

Kinematics

- Describes motion while ignoring the agents that caused the motion
- Today, we will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

Position

- Defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- The object's position is its location with respect to the frame of reference



UCF EXCEL

Displacement

- Defined as the change in position during some time interval
  - Represented as Δx

$$\Delta x = x_f - x_i$$

  - SI units are meters (m). Δx can be positive or negative
- Different than distance
- **Distance** is the length of a path followed by a particle

Average Speed

- **Average speed** is the distance traveled divided by the time that it took to travel that distance

  $$s_{avg} = \text{total distance / total time}$$

- The SI units are m/s. Dimensions are length/time [L/T]
- Speed is a scalar quantity
- Speed is the rate of change of position

Average Velocity

- **Average velocity** is the rate at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The dimensions are length / time [L/T]
- The SI units are m/s
- The average speed is not (necessarily) the magnitude of the average velocity
- Velocity is a vector quantity, so it has a direction associated with it

UCF EXCEL

Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity indicates what is happening at an instant of time
- The instantaneous velocity can be zero, positive, or negative

Instantaneous Speed

- **The instantaneous speed is the magnitude of the instantaneous velocity**
- **Remember that the average speed is <u>not</u> the magnitude of the average velocity**

UCF EXCEL NSF

Average Acceleration

- Acceleration is the rate of change of the velocity

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$$

- Dimensions are L/T^2
- SI units are m/s^2

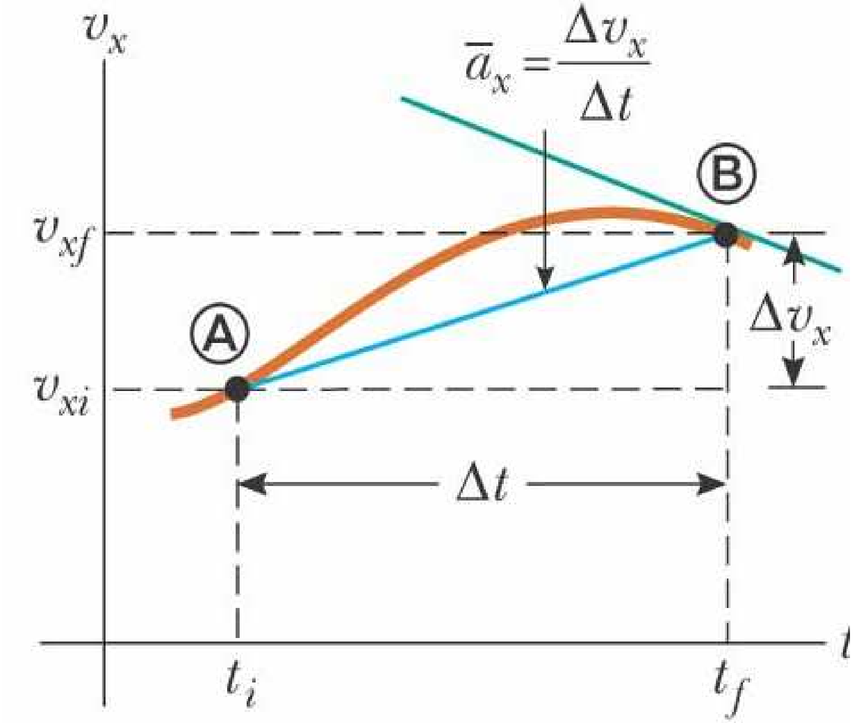
Instantaneous Acceleration

- **The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0**

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

Graphically

- The slope of the velocity vs. time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration



UCF EXCEL

Acceleration and Velocity

- When an object's velocity and acceleration are in the same direction, the object is speeding up
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down
- If velocity is constant, the acceleration is zero.

Freefall

- The most common constant acceleration is the acceleration that all freely falling bodies experience.
- Unfortunately, air resistance often confuses this fact.
- Galileo was the first to realize this and do experiments to confirm it.
  - The acceleration is directed towards the earth, regardless of the initial motion.
  - When an object is thrown upward or downward, it is falling freely once it is released.

Apollo 15

QuickTime™ and a
Microsoft Video 1 decompressor
are needed to see this picture.

Apollo 15 astronaut David Scott tested Galileo's Law of Falling Bodies on the surface of the moon.

UCF EXCEL

Freefall Acceleration

- The value of this acceleration is approximately 9.80 m/s^2 (32 ft/s^2)
- The symbol that we use is *g*
- Typically, we choose down, or the direction toward the earth, to be negative. Therefore $g = -9.8\ m/s^2$
- If we were to go to another planet, the value of *g* would be different (if the mass and/or the radius of the planet is different from that of Earth)

UCF EXCEL

Speed 2 Example

	time	speed in knots	speed in m/s	deceleration
1	0	10.5	5.25	
2		10.2	5.10	
3		9.6	4.80	
4		7.3	3.65	
5		6.9	3.45	
6		5.3	2.65	
7		2.0	1.00	
8		1.3	0.65	
9		0	0	

1 knot ≈ 0.5 *m/s*

UCF EXCEL

Speed 2 Clip

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

Calculation

	Δ time	speed in knots	speed in m/s	deceleration
1	0	10.5	5.25	
2		10.2	5.1	#DIV/0!
3		9.6	4.8	#DIV/0!
4		7.3	3.65	#DIV/0!
5		5.8	2.9	#DIV/0!
6		5.3	2.65	#DIV/0!
7		2	1	#DIV/0!
8		1.3	0.65	#DIV/0!
9		0	0	#DIV/0!

Tango and Cash

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.